Quantifying uncertainties in light-ion reactions using EFT and Bayesian methods

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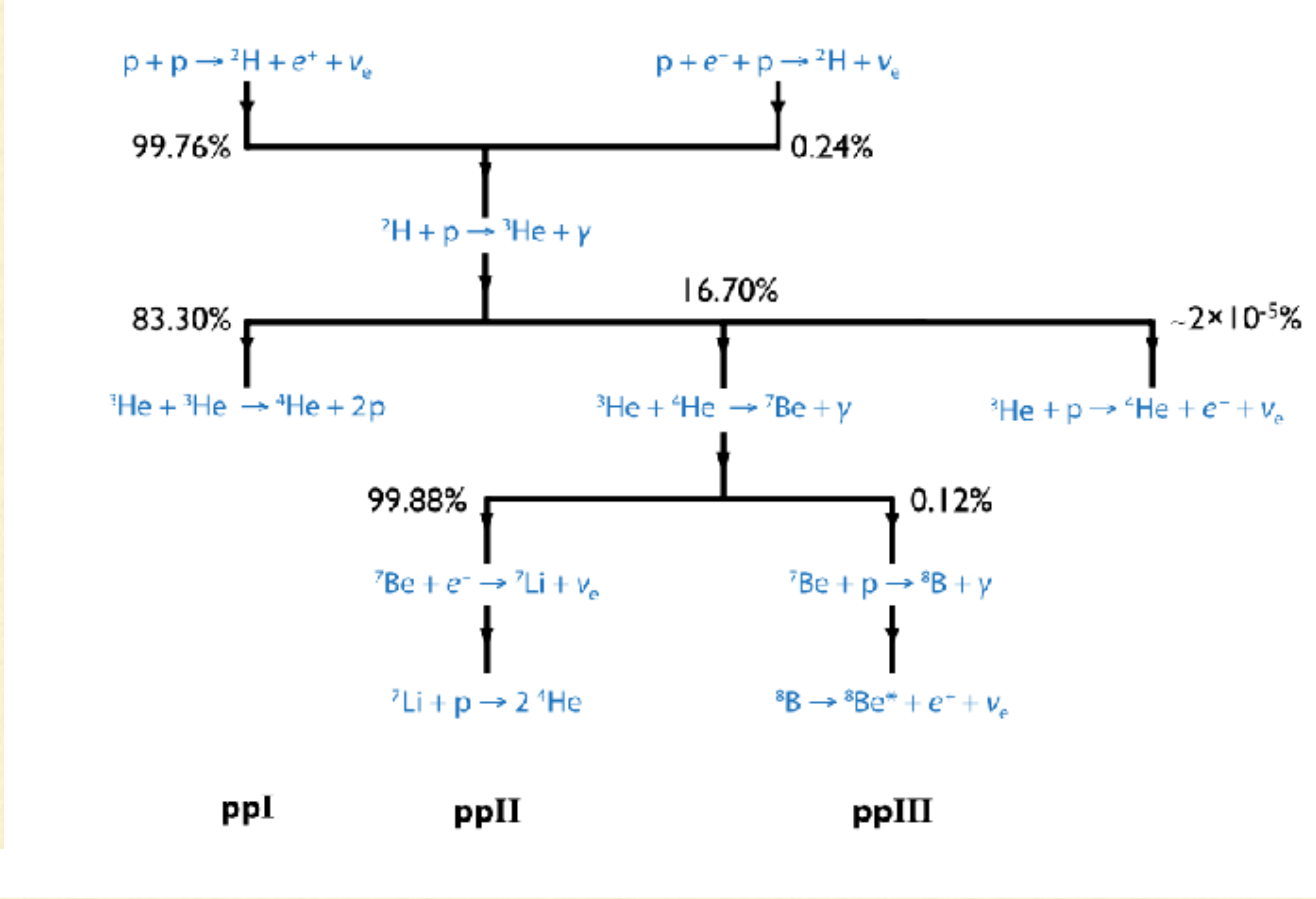
Why is ^{3}He(^{4}He,γ) important?

Adelberger et al., Rev. Mod. Phys. 83, 195 (2011)

- Accurate knowledge of ^{3}He(^{4}He,γ) needed to reliably predict amount of ^{7}Be in the Sun
- Therefore key for prediction of ^{8}B solar neutrino flux
- BBN implications, but I will not discuss those here



Building a good extrapolant

$$\mathscr{M}(E) \propto \int dr\, A_1 \exp(-\gamma_1 r)\left(1 + \frac{1}{\gamma_1 r}\right) r u_E(r)$$

Dominated by inter-nucleus separations outside V(r)

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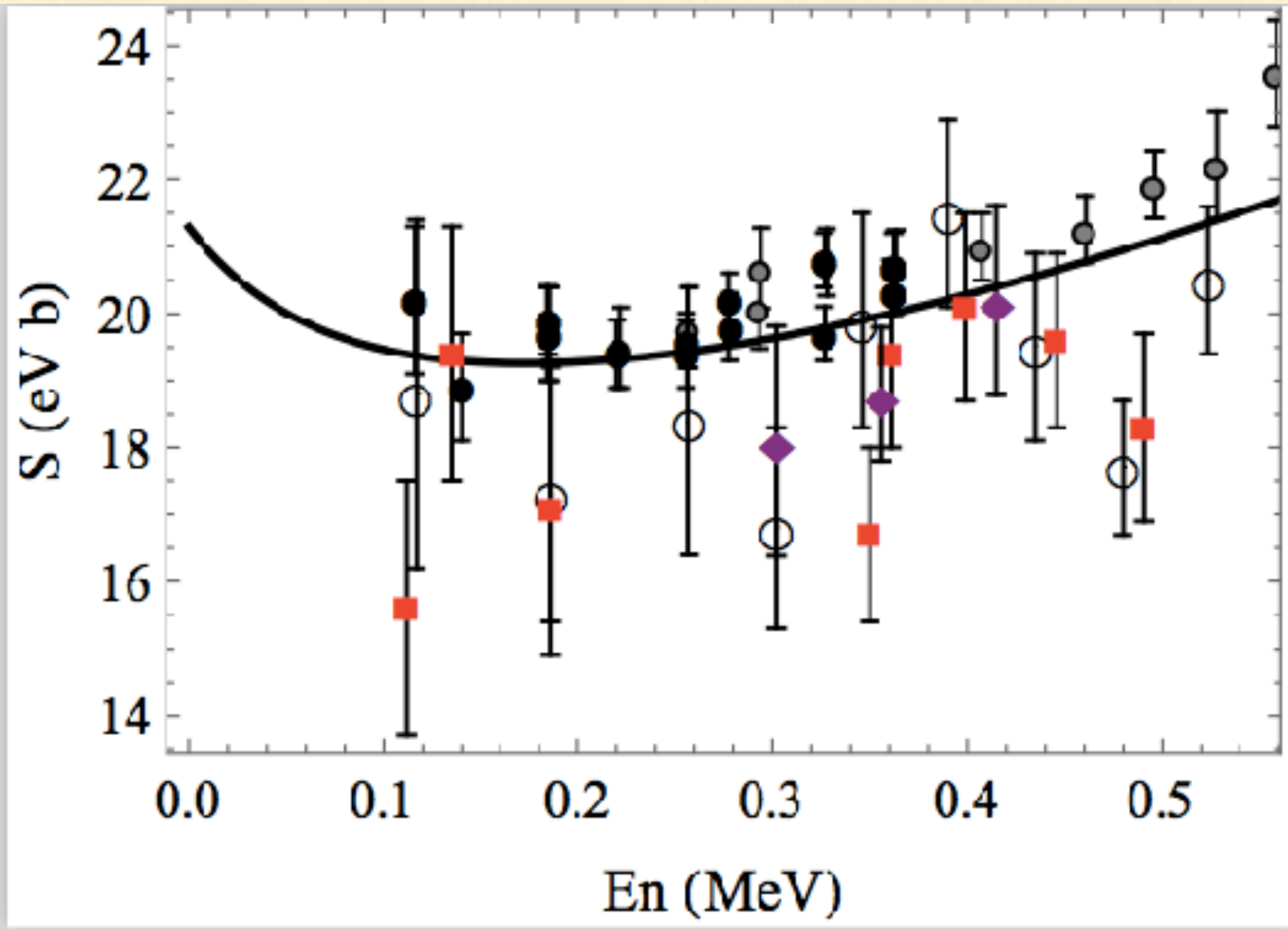

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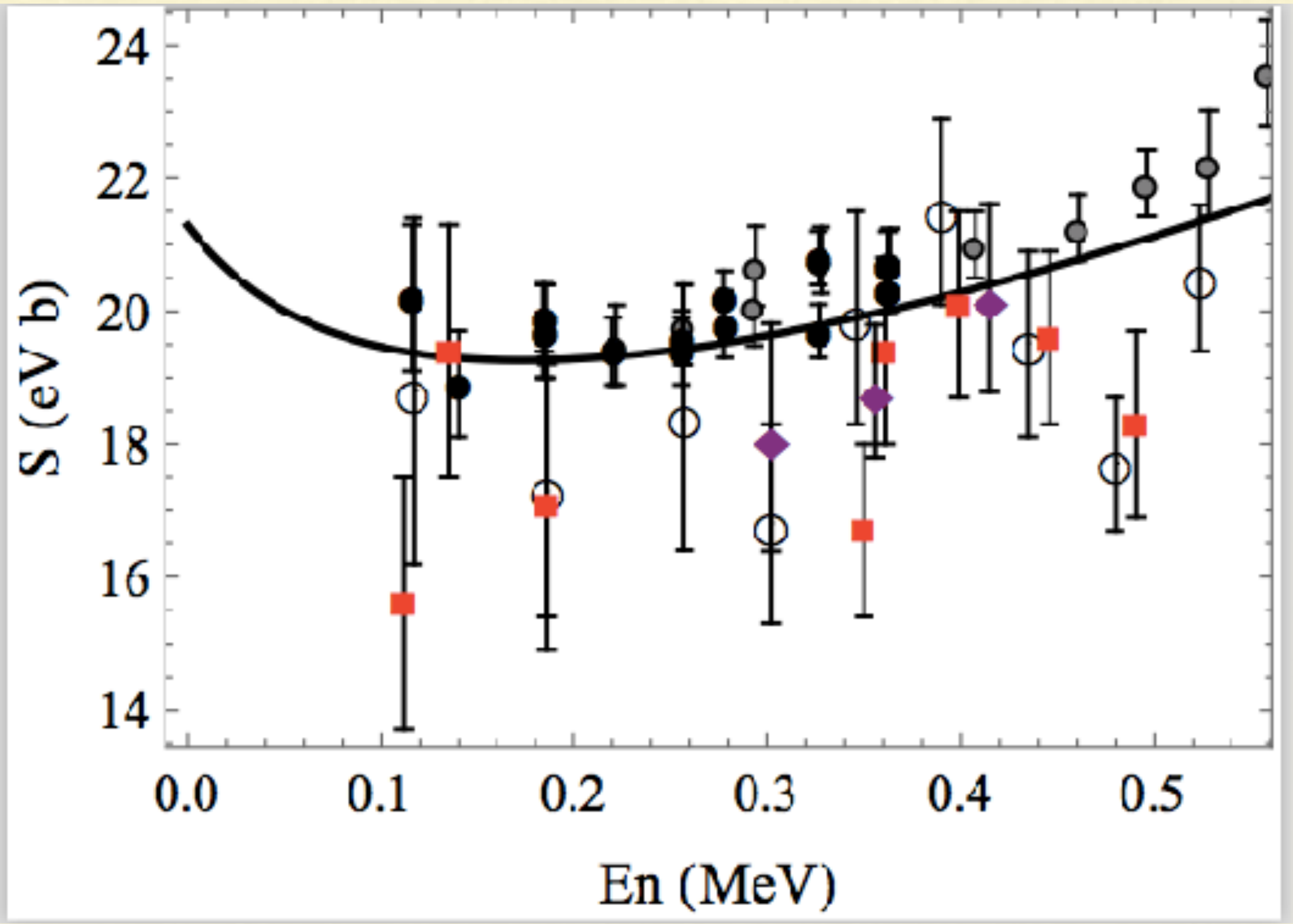

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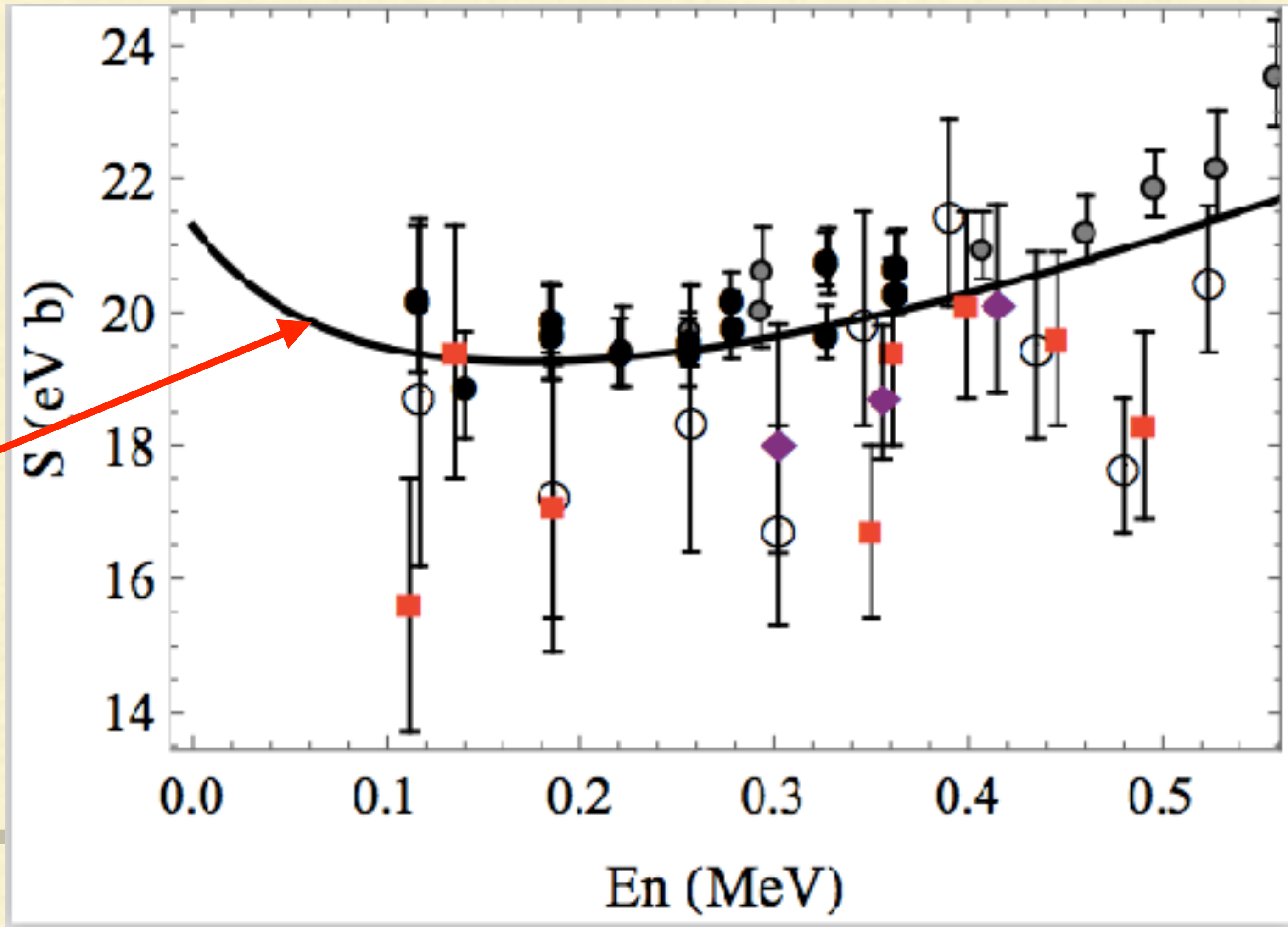

Bound state (ANC &γ_1)
+ Coulomb

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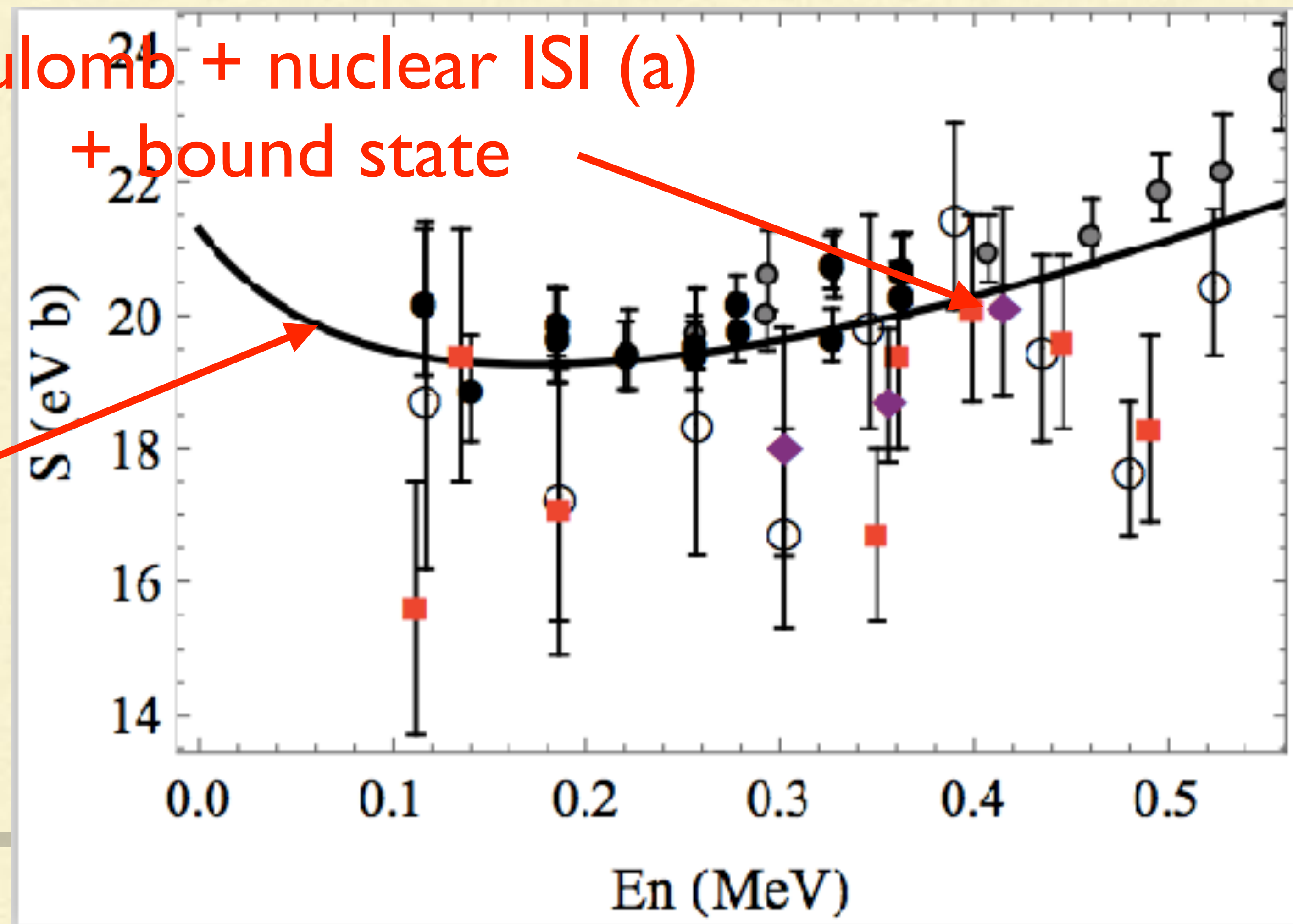
Coulomb + nuclear ISI (a) + bound state



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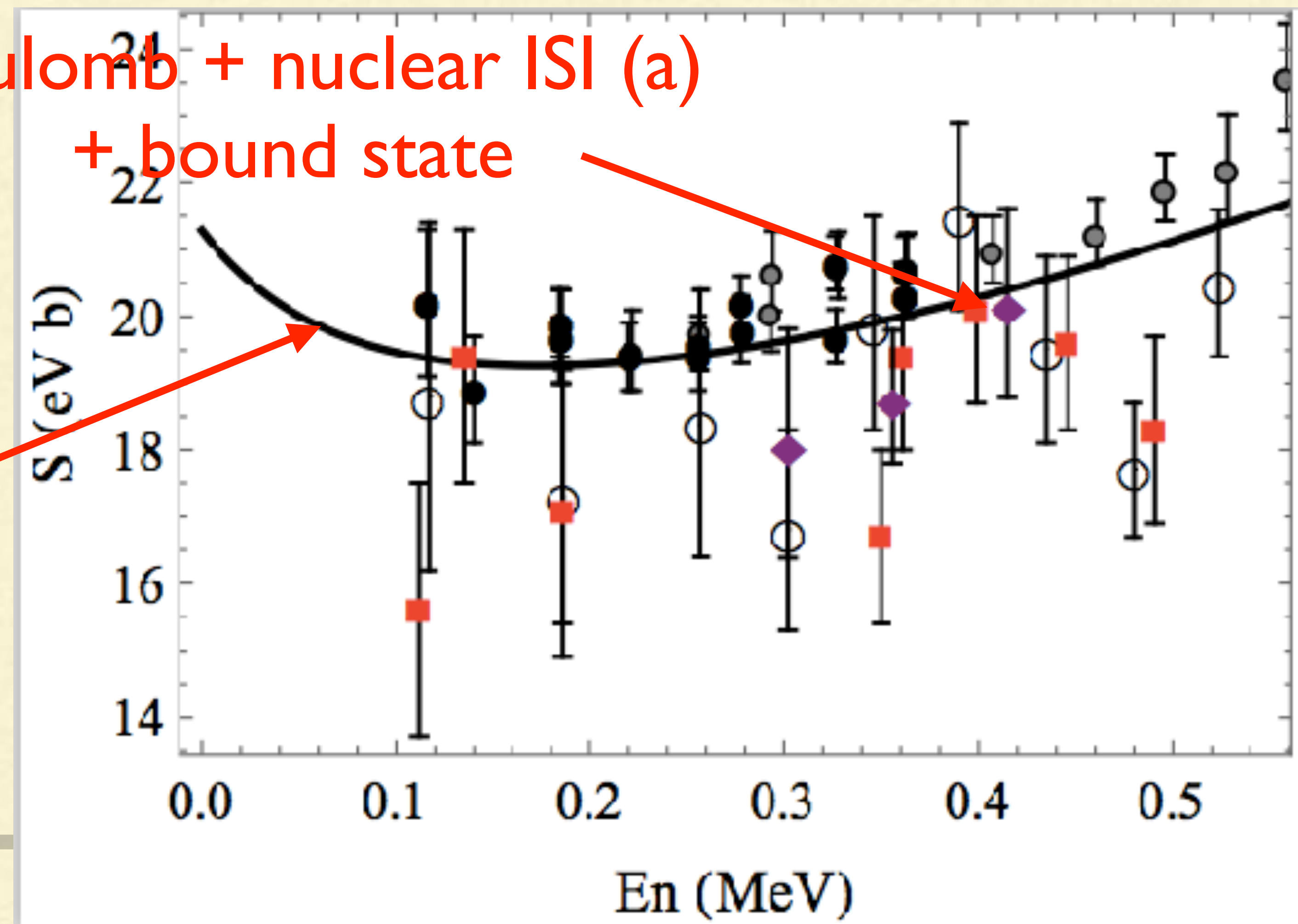
- Extrapolation is not a polynomial: non-analyticities in p/k_C, p/γ_1, and p a
- Sub-leading polynomial behavior in E/E_{core} corrects for what happens inside V(r)

Coulomb + nuclear ISI (a) + bound state



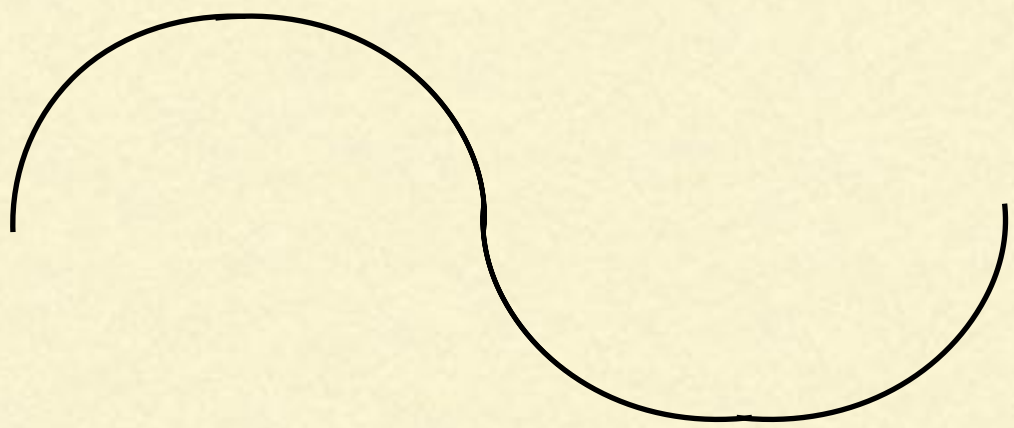
Bound state (ANC & γ_1) + Coulomb

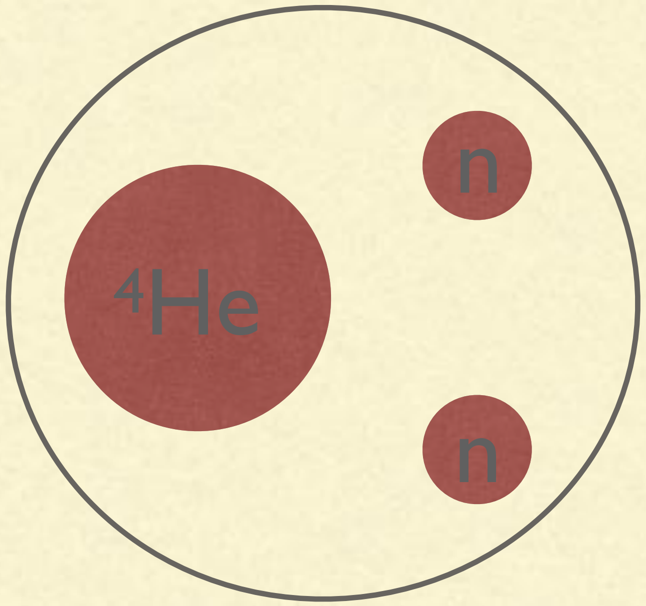
Outline

- $^3He + {}^4He \rightarrow {}^7Be + \gamma$ is an important extrapolation problem
- How Halo Effective Field Theory can help
- From S-factor and branching-ratio data to Halo EFT parameters
- From scattering results to Halo EFT parameters
- Fully realizing the benefits of the EFT: EFT error estimates
- Parameter estimation with EFT error estimates
- Summary and Future Work

Halo EFT

$\lambda \gg R_{core};\ \lambda \lesssim R_{halo}$





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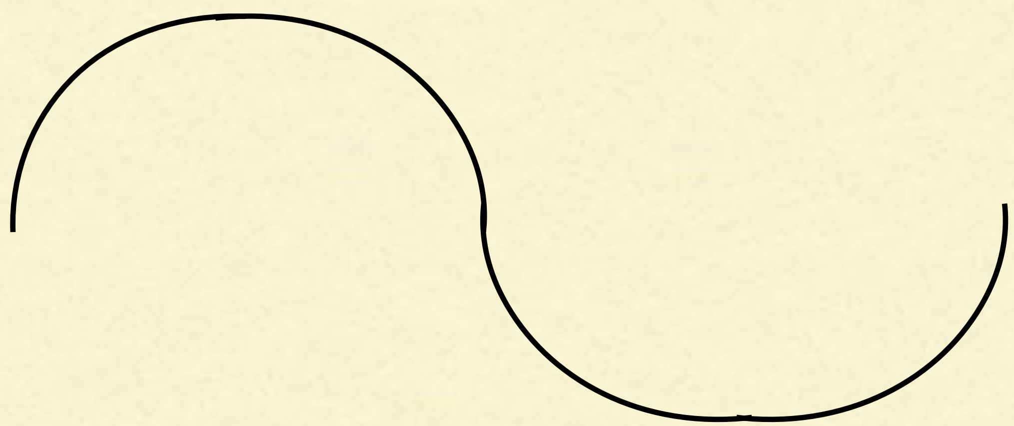
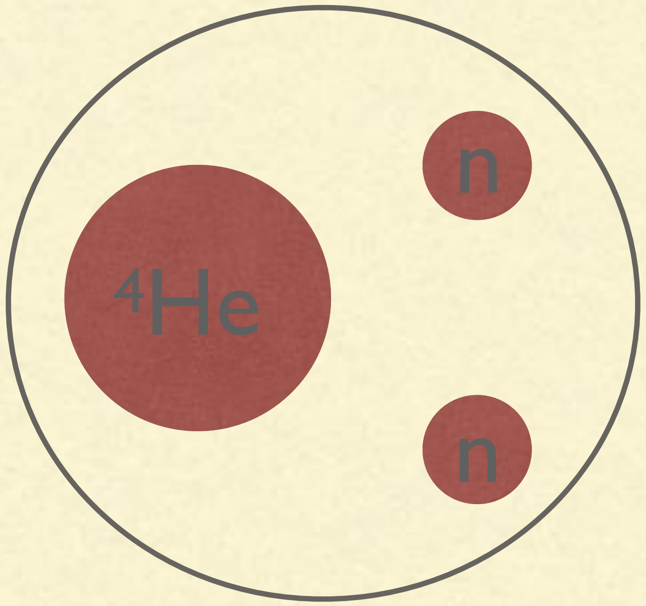




- Define $R_{halo}=\langle r^2\rangle^{1/2}$. Seek EFT expansion in R_{core}/R_{halo}. Valid for $\lambda \lesssim R_{halo}$
- Typically $R \equiv R_{core} \sim 2$ fm. And since $\langle r^2\rangle$ is related to the neutron separation energy we are looking for systems with neutron separation energies less than 1 MeV
- By this definition the deuteron is the lightest halo nucleus, and the pionless EFT for few-nucleon systems is a specific case of halo EFT

Lagrangian: shallow S- and P-states

$$
\begin{aligned}
\mathcal{L} \quad = \quad & c^\dagger \left(i\partial_t + \frac{\nabla^2}{2M} \right) c + n^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} \right) n \\
& + \sigma^\dagger \left[\eta_0 \left(i\partial_t + \frac{\nabla^2}{2M_{nc}} \right) + \Delta_0 \right] \sigma + \pi_j^\dagger \left[\eta_1 \left(i\partial_t + \frac{\nabla^2}{2M_{nc}} \right) + \Delta_1 \right] \pi_j \\
& - g_0 \left[\sigma n^\dagger c^\dagger + \sigma^\dagger n c \right] - \frac{g_1}{2} \left[\pi_j^\dagger (n \, i\overleftrightarrow{\nabla}_j \, c) + (c^\dagger \, i\overleftrightarrow{\nabla}_j \, n^\dagger) \pi_j \right] \\
& - \frac{g_1}{2} \frac{M - m}{M_{nc}} \left[\pi_j^\dagger \, i\overrightarrow{\nabla}_j \, (nc) - i\overleftrightarrow{\nabla}_j \, (n^\dagger c^\dagger) \pi_j \right] + \ldots \, ,
\end{aligned}
$$

- c, n: "core", "neutron" fields. c: boson, n: fermion.
- σ, π_j: S-wave and P-wave fields
- Minimal substitution generates leading EM couplings
- Additional EM couplings at sub-leading order

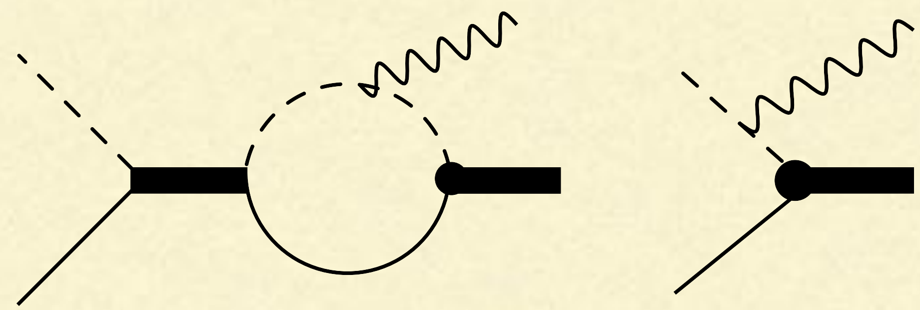
p-wave bound states and capture thereto

Hammer & DP, NPA (2011)

- At LO p-wave 1n halo described solely by its ANC and binding energy

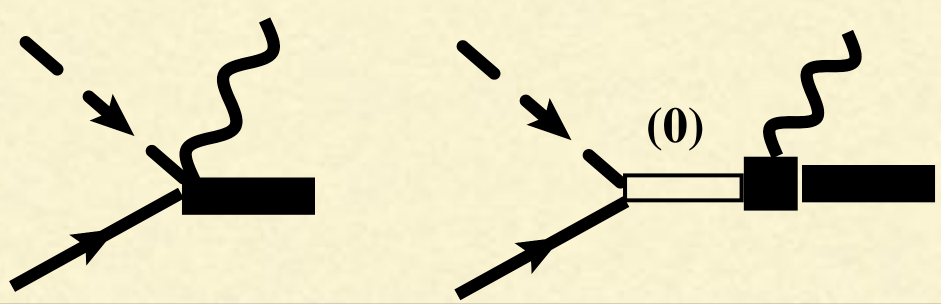
$$u_1(r) = A_1 \exp(-\gamma_1 r)\left(1 + \frac{1}{\gamma_1 r}\right) \qquad \gamma_1 = \sqrt{2 m_R B}$$

- Capture to the p-wave state proceeds via the one-body E1 operator: "external direct capture"



$$\mathrm{E1} \propto \int dr\, u_1(r) r(\cos(kr) + \sin(kr)\ \cot\delta); k\cot\delta \text{ from ERE}$$

- NLO: piece of the amplitude representing capture at short distances, represented by a contact operator⇒there is an LEC that must be fit



$^{3}He + {}^{4}He \rightarrow {}^{7}Be + \gamma_{E1}$ at LO in Halo EFT

Zhang, Nollett, DP, JPG (2019), cf. Rupak, Higa, Vaghani, EPJA (2018)

- In this system $R_{core} \sim 1.5$ fm, $R_{halo} \sim 3$ fm

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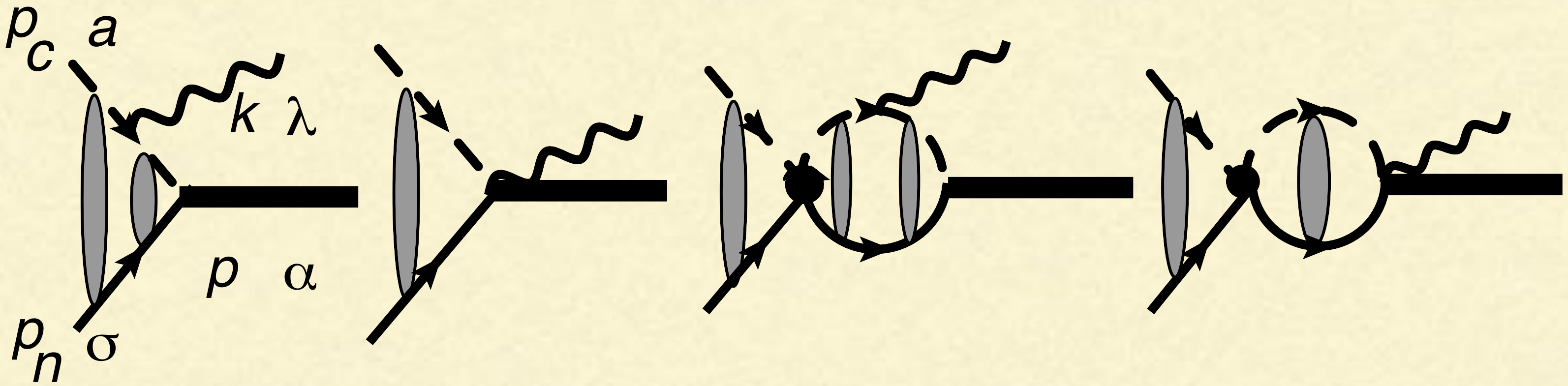
- In this system R_{core}~1.5 fm, R_{halo} ~3 fm
- Also need to include Coulomb interactions non-perturbatively: $k_C=Q_cQ_n\alpha_{EM}M_R$=17 MeV; a~10s of fm, both ~R_{halo}



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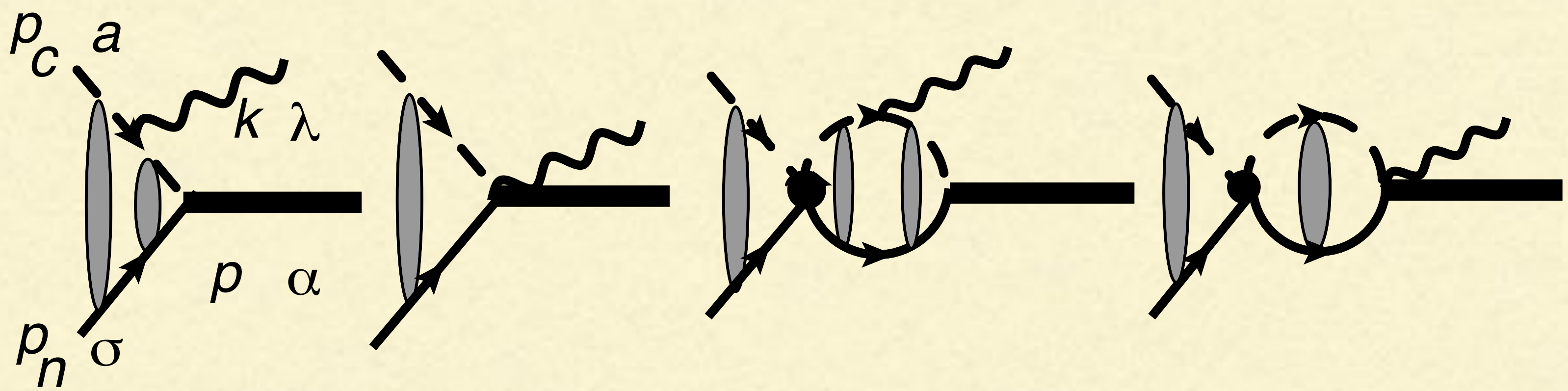

- Scattering wave functions are linear combinations of Coulomb wave functions F_0 and G_0. Bound state wave function=the appropriate Whittaker function.

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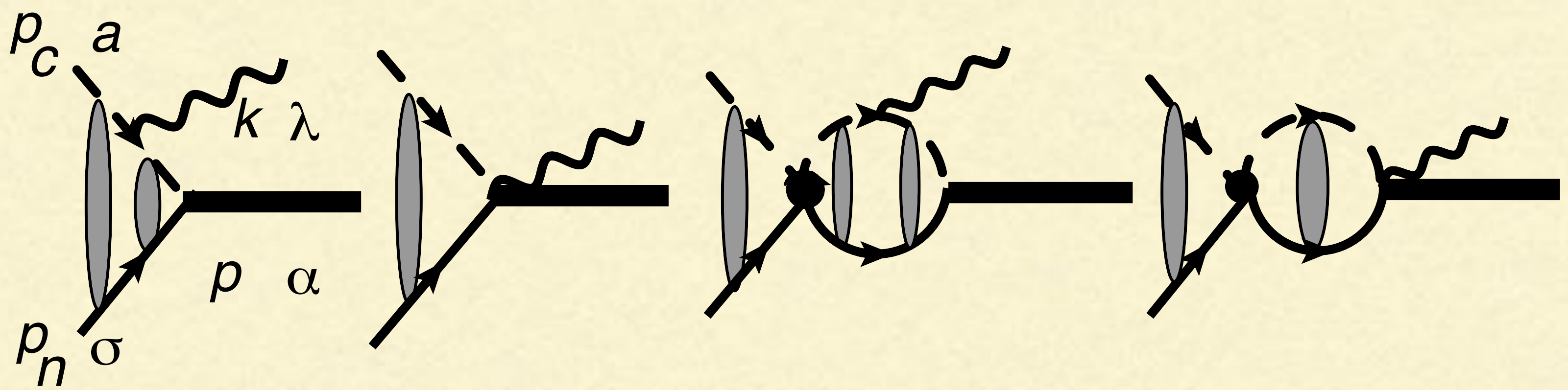

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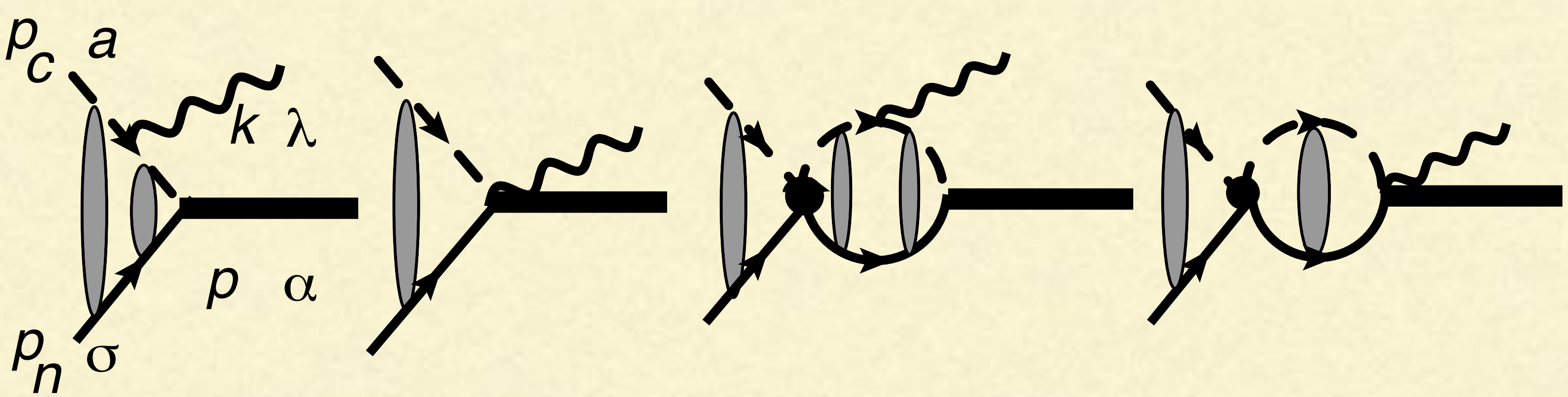

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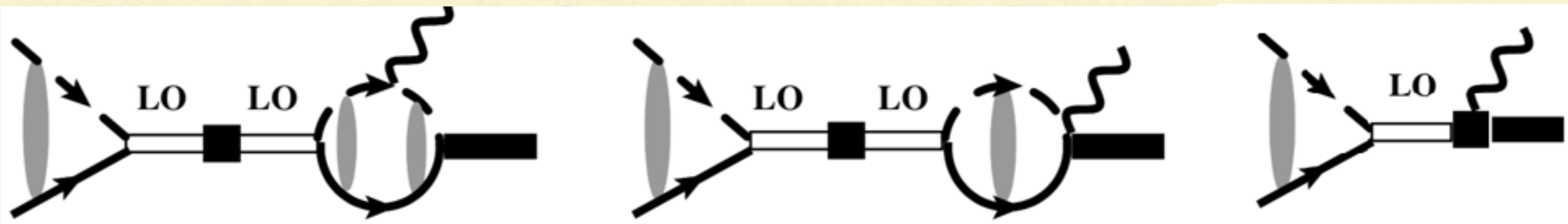
- Can also predict capture to the excited 1/2$^-$ in ^{7}Be

Three parameters at leading order

Additional ingredients at NLO



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Three more parameters at NLO

- Effective range (can add shape parameter which enters at N^{3}LO)
- LECs associated with contact interaction, $\bar{L}$ and $\bar{L}_*$
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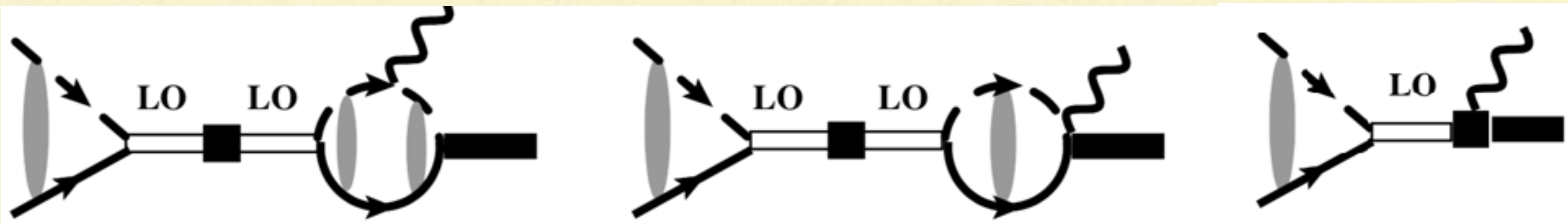

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Data for $^3He + {}^4He \rightarrow {}^7Be + \gamma_{E1}$

- 59 S-factor data below 2 MeV
    - Seattle (S)
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- In general use activation data, to avoid photon emission asymmetry systematic; recoil data from Erna; prompt measurements from Notre Dame
- Deal with CMEs by introducing six additional parameters, ξ_i
- Plus 32 branching-ratio data: CMEs assumed absent there

Building the pdf

- χ^2 needs to include cross-section and branching-ratio data

$$\chi^2 \equiv \sum_J^{N_{\exp}} \left\{ \sum_{j=1}^{N_{s,J}} \frac{\left[(1-\xi_J)S(\overrightarrow{g};E_{Jj}) - D_{Jj}\right]^2}{\sigma_{Jj}^2} + \frac{\xi_J^2}{\sigma_{c,J}^2} \right\} + \sum_{l=1}^{N_{br}} \frac{\left[Br(\overrightarrow{g};E_l) - \tilde{D}_l\right]^2}{\sigma_{br,l}^2}$$

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- Mild Bayesian priors:
  - Independent gaussian priors for ξ_i, centered at zero and with width=CME
  - Other EFT parameters, a, r, L, and two ANCs assigned flat priors, corresponding to natural ranges
- Probability $e^{-\chi^2/2}$ sampled using Markov Chain Monte Carlo

$^3He(^4He,\gamma)$ results

Zhang, Nollett, DP, JPG (2019)
cf. Higa, Rupak, Vaghani, EPJA (2018)

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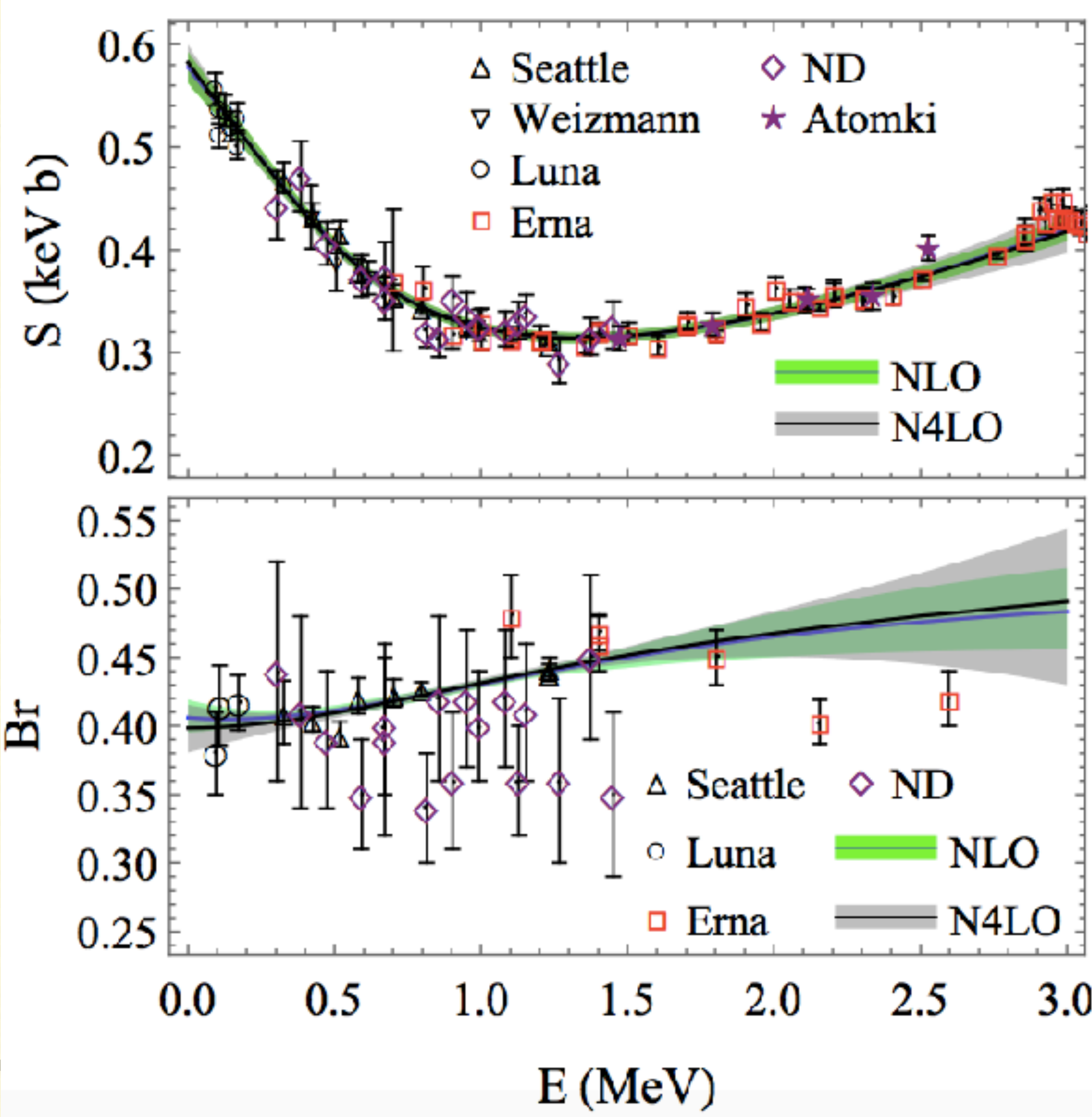

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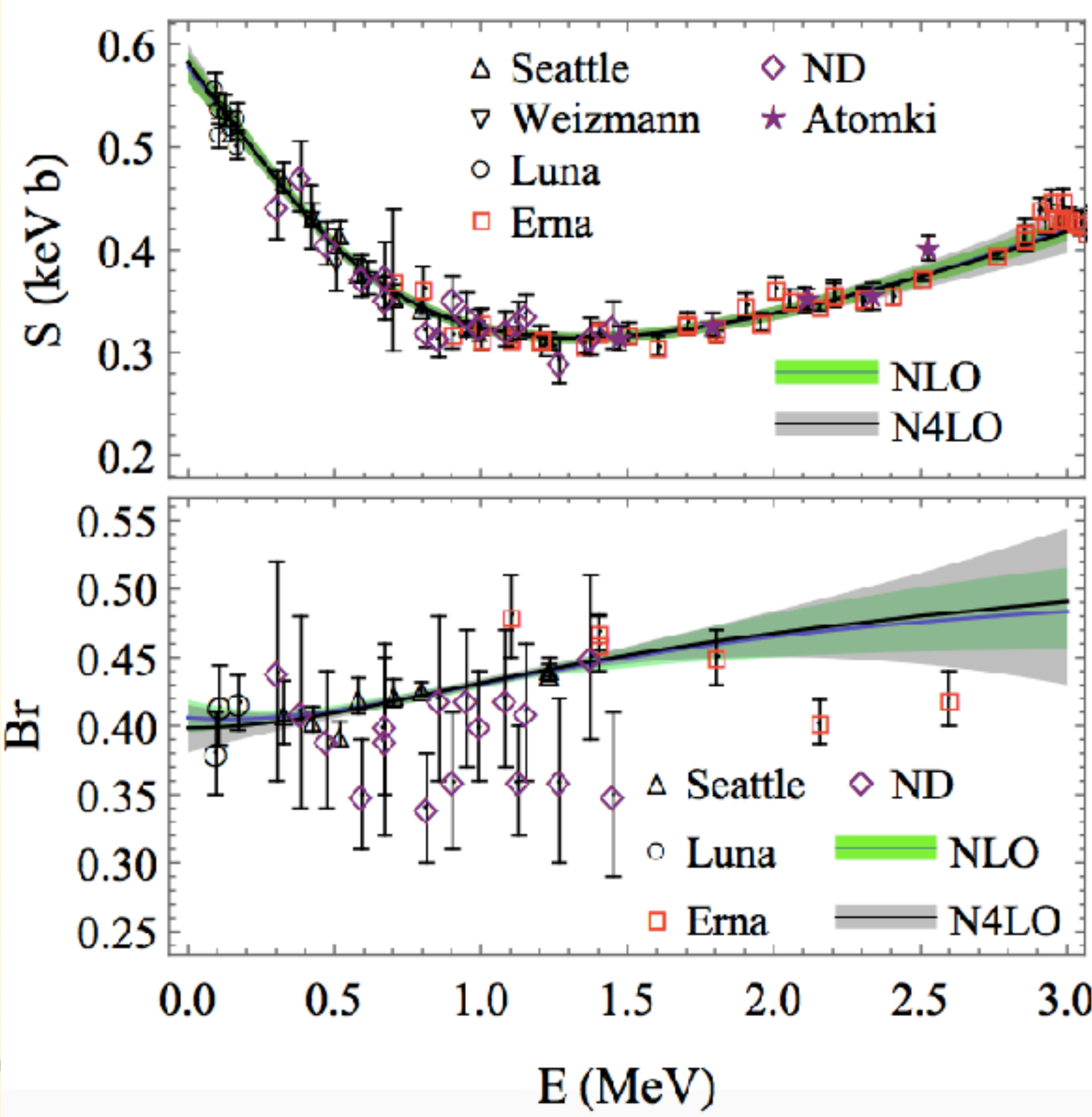
- Distribution peaks at $\chi^2 = 82$



^{3}He(^{4}He,ɣ) results

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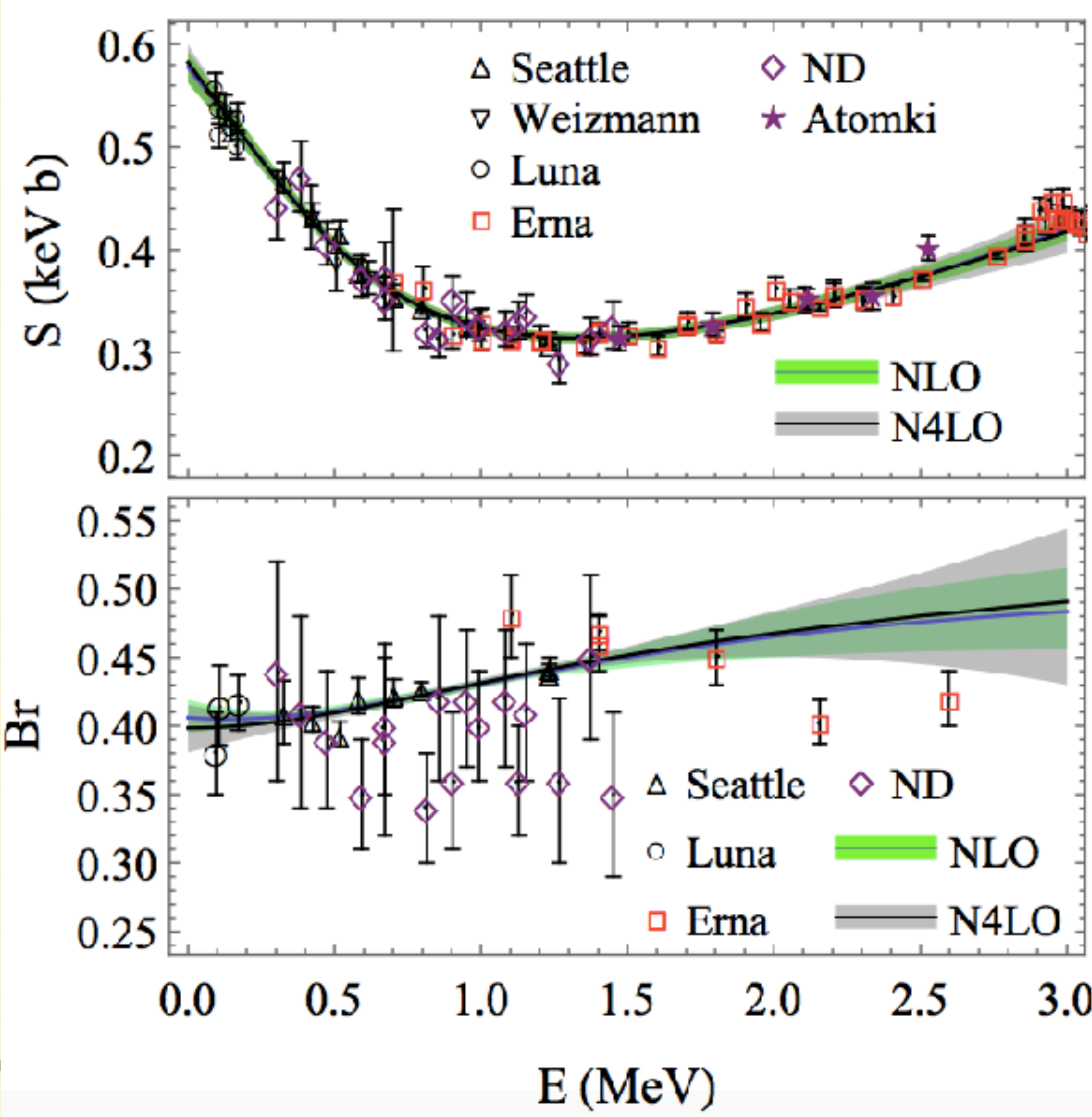
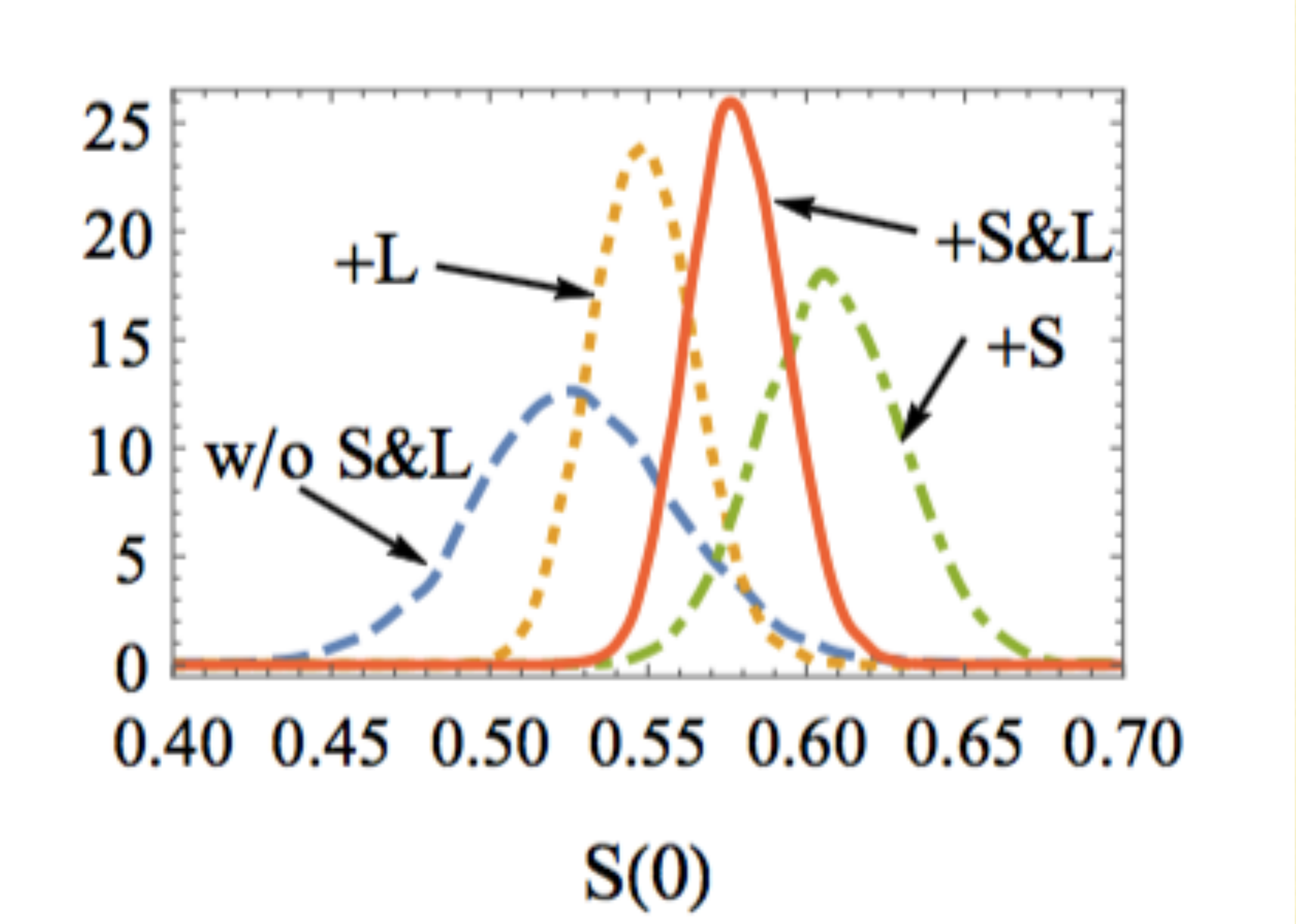
- Distribution peaks at $\chi^2 = 82$
- Bayesian evidence ratio≅6 for NLO cf. N^{4}LO



Impact of different data sets

- Floating data within quoted CME crucial for achieving data consistency
- Pdf gets narrower when either of the precise, low-energy data sets are included
- Seattle data push S(0) to higher values, but still possible to find concordance between Seattle, Luna, and older data



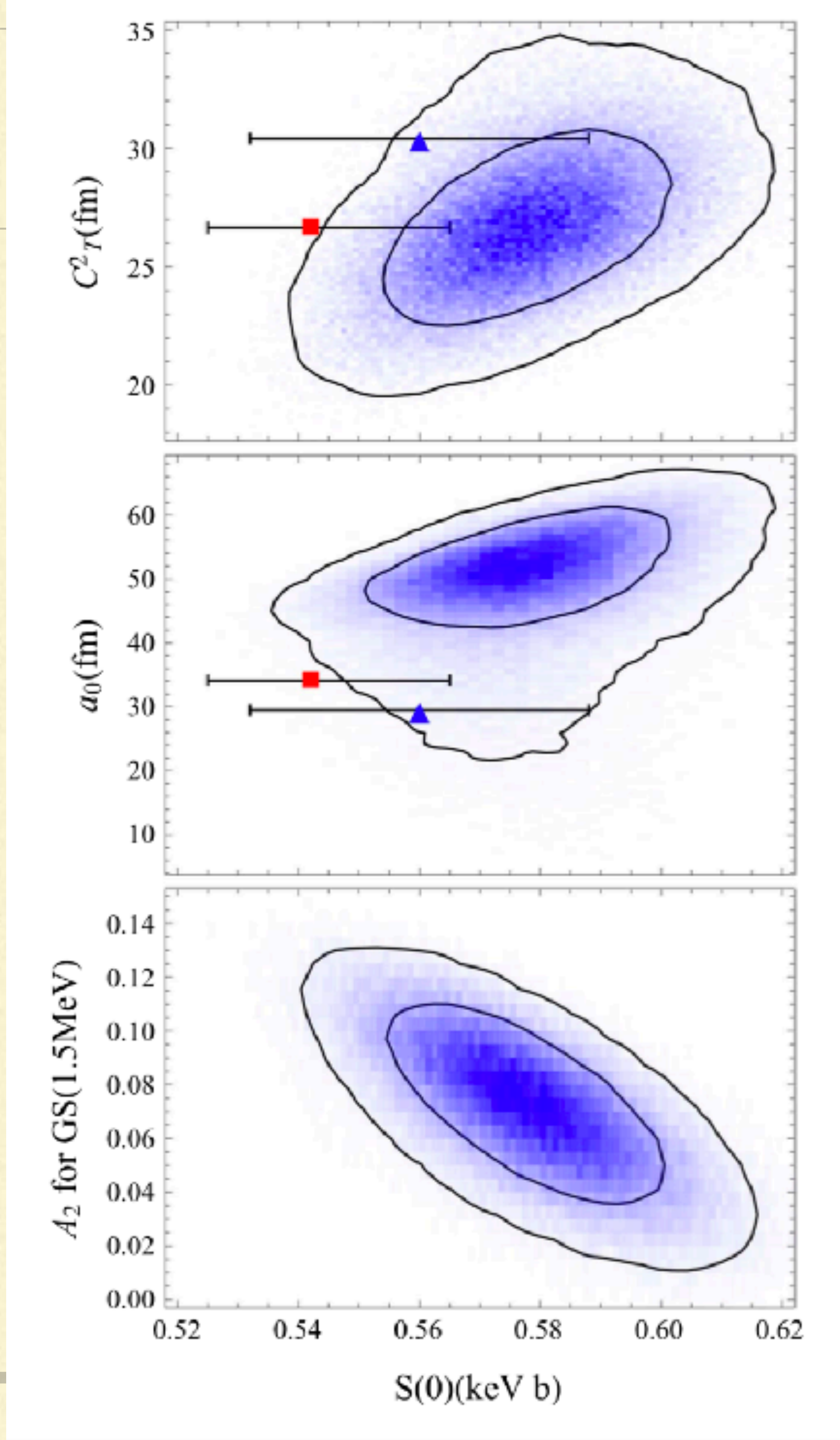
S(0) and its correlants

$$S(0) = 0.578^{+0.015}_{-0.016} \text{ keV b}$$

cf. SFII: $S(0) = 0.56 \pm 0.03 \text{ keV b}$

$$Br(0) = 0.406^{+0.013}_{-0.011}$$

Mostly consistent with other analyses, but 1.5σ higher than that of deBoer et al.



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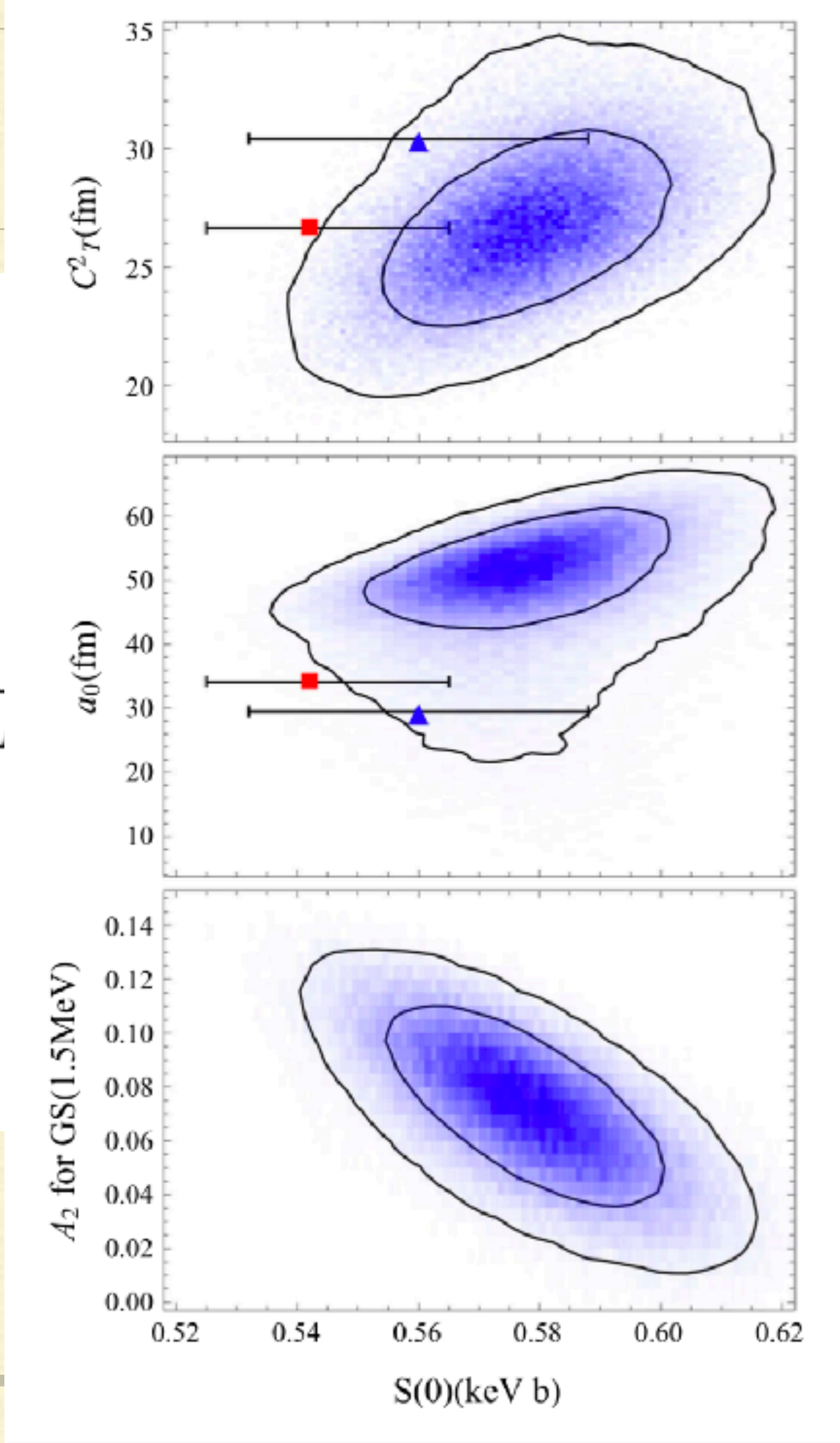
How to do better on S(0)?

1. Measure $P_2(\cos\Theta)$ dependence
2. Tight constraints on scattering parameters from capture data alone



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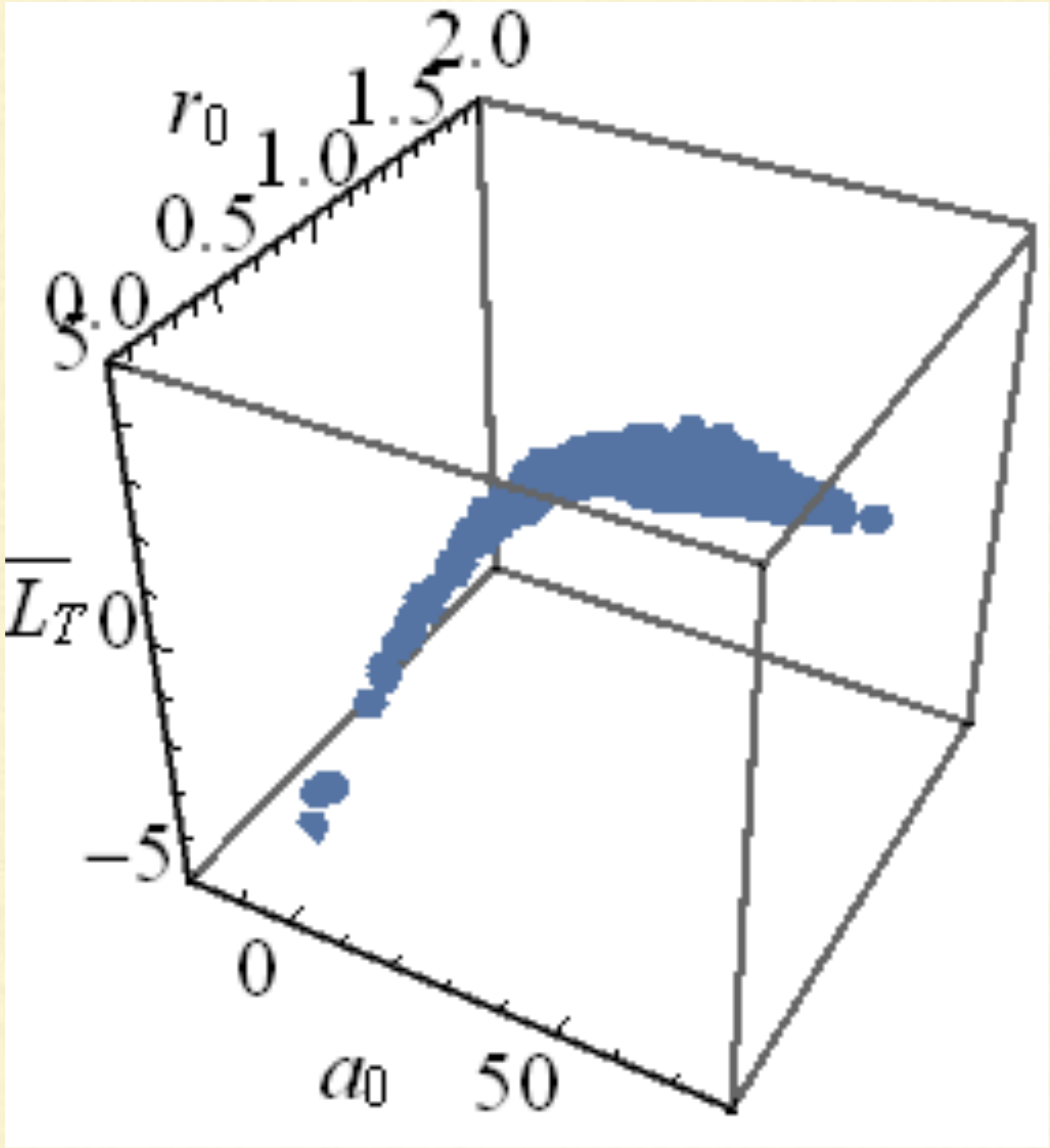

How to do better on S(0)?

1. Measure $P_2(\cos\Theta)$ dependence
2. Tight constraints on scattering parameters from capture data alone

S(0) and its correlants

$$S(0) = 0.578^{+0.015}_{-0.016} \text{ keV b}$$

cf. SFII: $S(0) = 0.56 \pm 0.03 \text{ keV b}$

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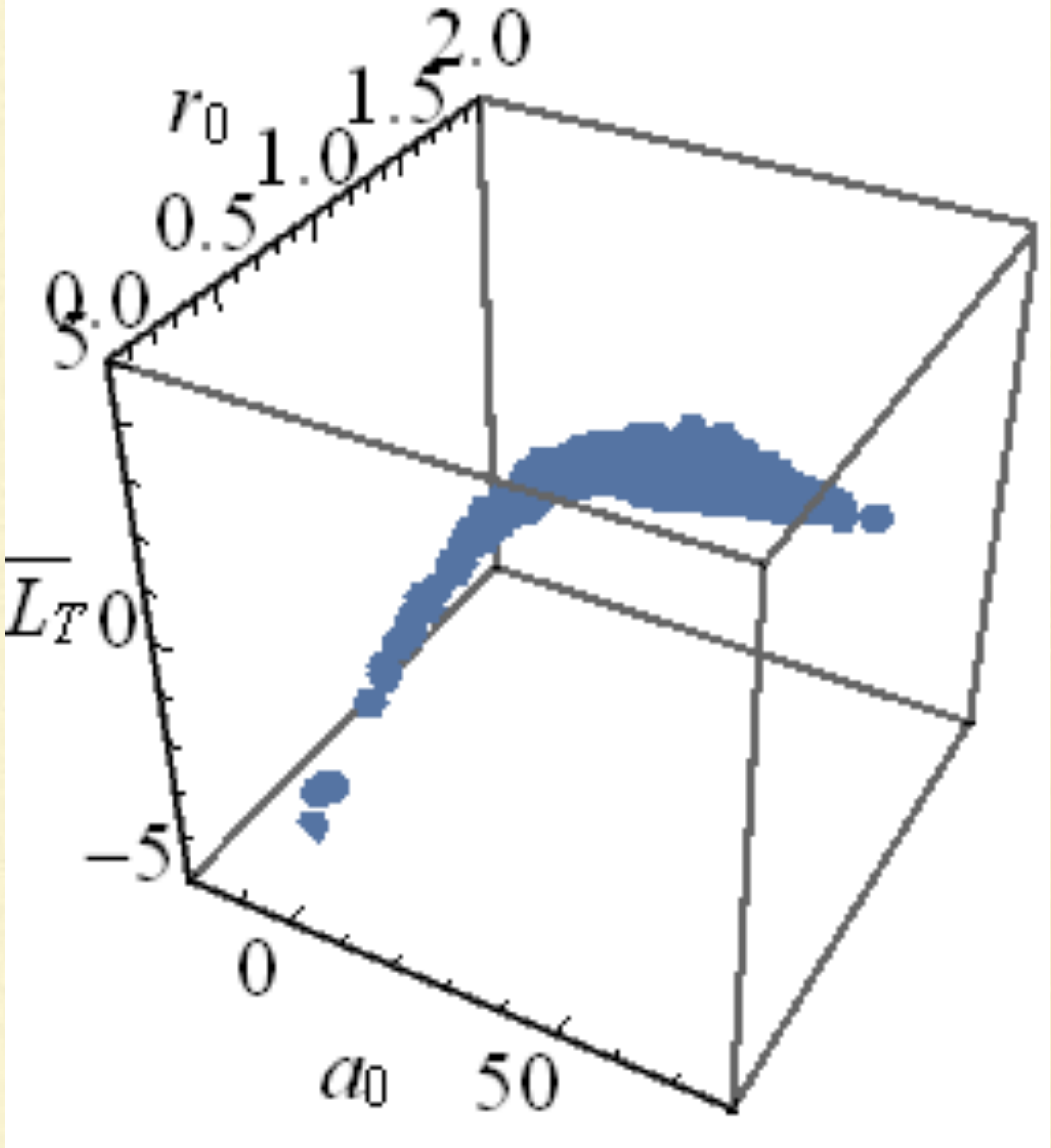
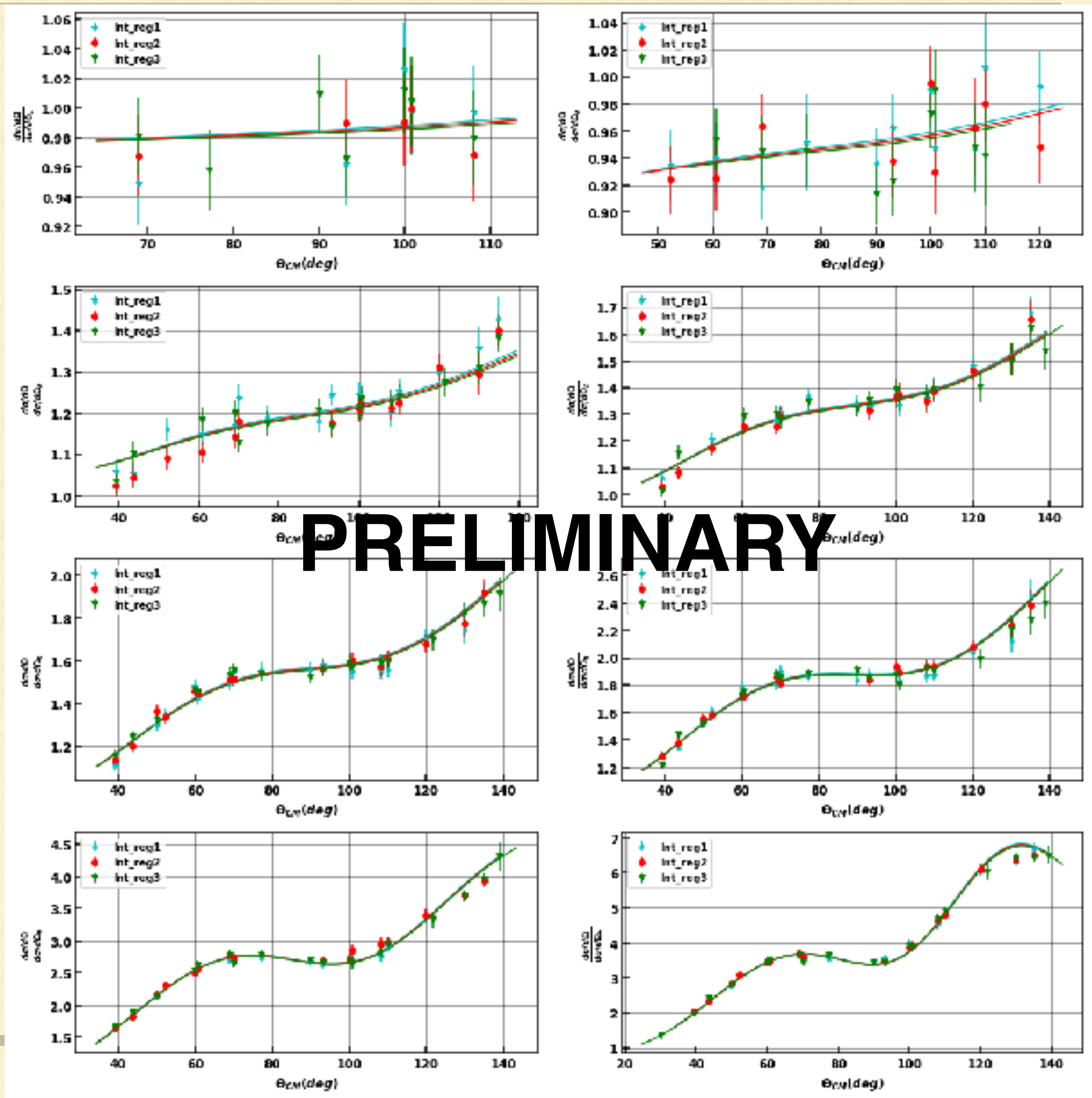


$$a_0 = 50^{+7}_{-6} \text{ fm}$$

EFT treatment of ^{3}He + ^{4}He scattering

Mahesh Poudel and DP, in preparation

- Analyze recent Paneru et al. TRIUMF data using Halo EFT to N3LO
- S-waves: a_0, r_0, P_0
- P-waves: a_1, r_1, P_1 ($\Leftrightarrow E_{7Be}$, ANC, P_1)
- F-waves: Resonance at E_{cm}=2.98 MeV with fitted Γ (R-matrix form)
- Also include Boykin et al. A_y data and compare to older scattering data set of Barnard



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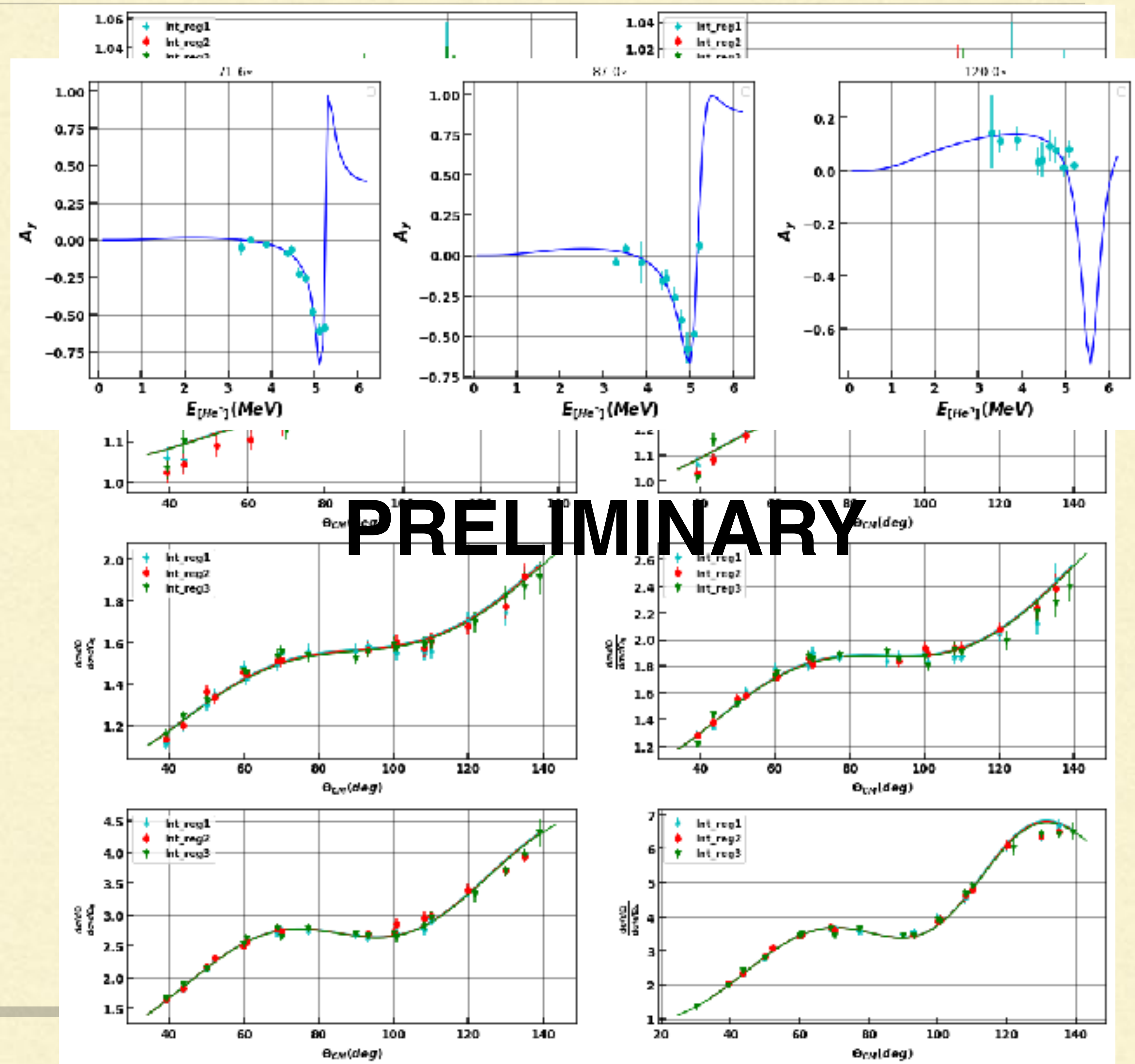
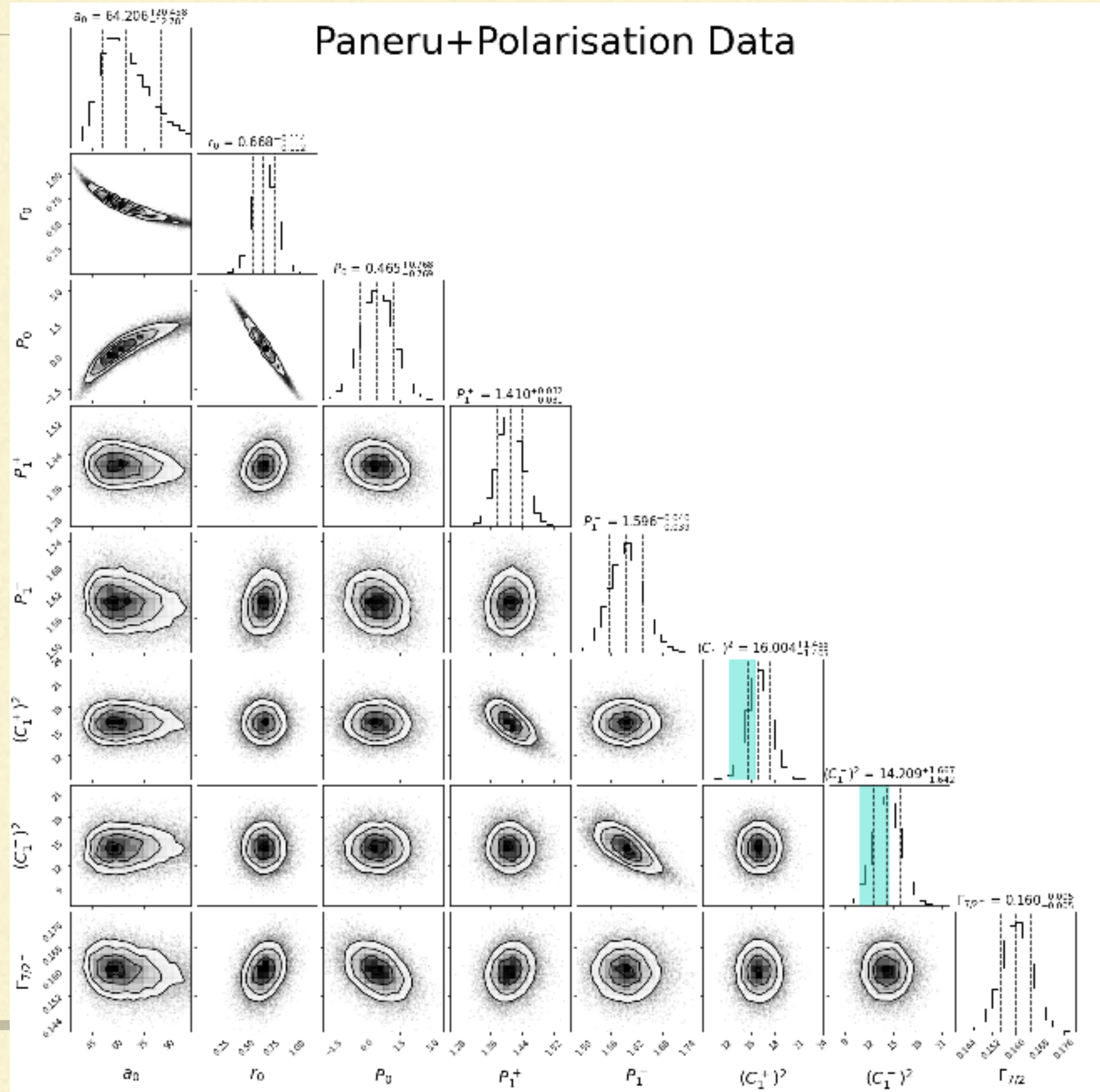


ERT parameters from scattering data

- Imposed prior on ANCs from capture data, so not *solely* from scattering data

- Consistent values: $C_1^{+2} = 16.0 \pm 1.4$ fm; $C_1^{-2} = 14.2 \pm 1.6$ fm

- $a_0 = 64^{+20}_{-13}$ fm cf. $a_0 = 50^{+7}_{-6}$ fm from capture (NLO analysis)



Outline

- ^{3}He + ^{4}He$\rightarrow$^{7}Be + γ is an important extrapolation problem √
- How Halo Effective Field Theory can help √
- From S-factor and branching-ratio data to Halo EFT parameters √
- From scattering results to Halo EFT parameters √
- Fully realizing the benefits of the EFT: EFT error estimates
- Parameter estimation with EFT error estimates
- Summary and Future Work

An EFT expansion in pictures

- General EFT series for observable to order k: $y = y_{\rm ref} \sum_{n=0}^{k} c_n Q^n$
- In ChiEFT $Q = \frac{(p, m_\pi)}{\Lambda_b}$; In Halo EFT $Q = \frac{(p, \gamma_1)}{\Lambda_b}$; $\Lambda_b \approx 150$ MeV

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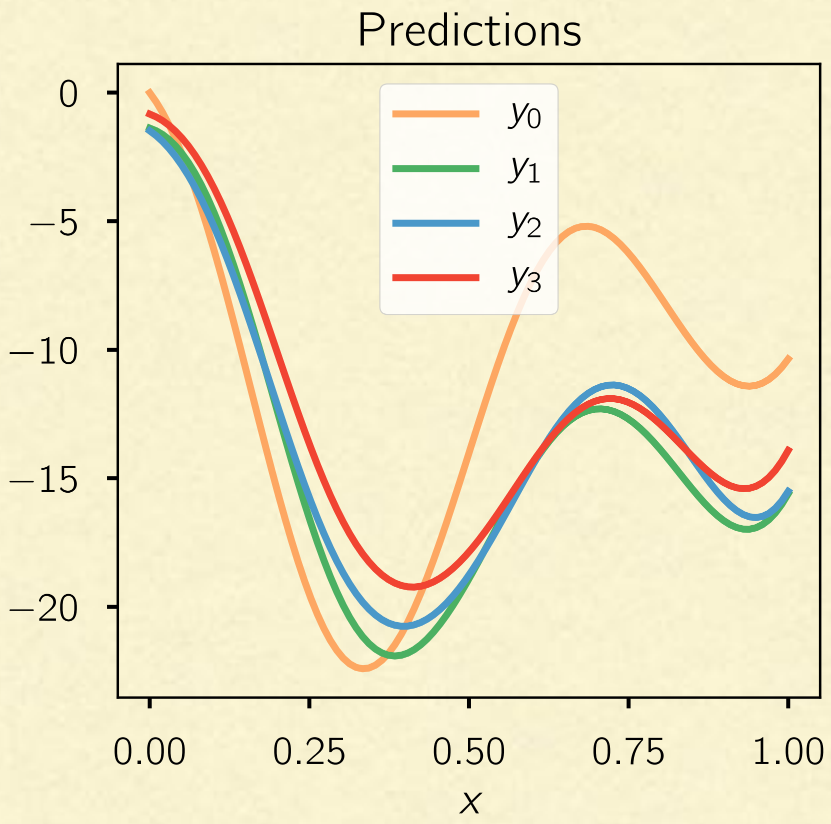

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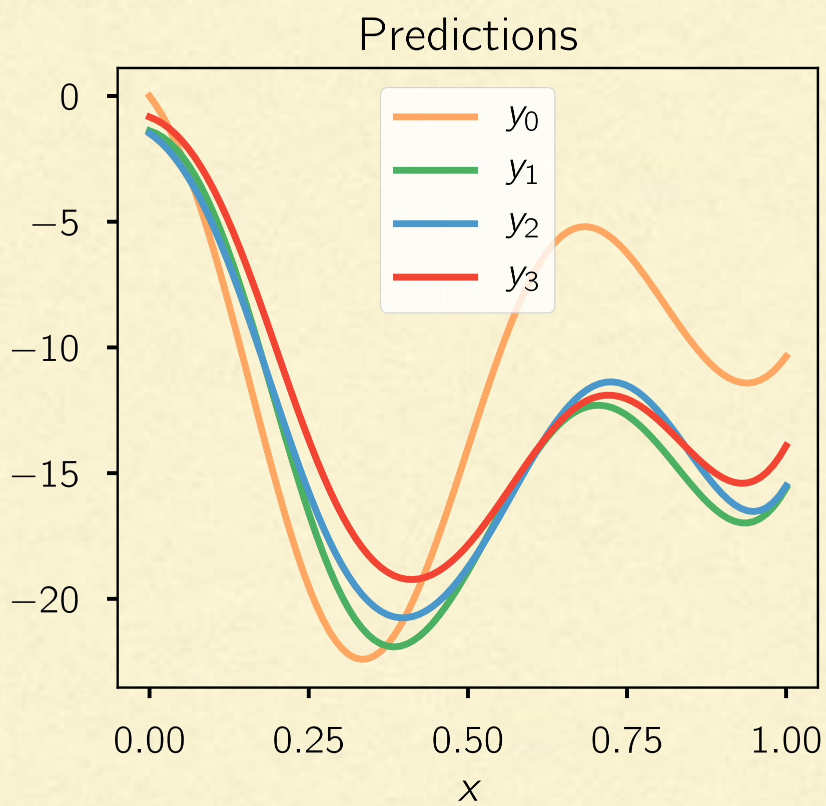

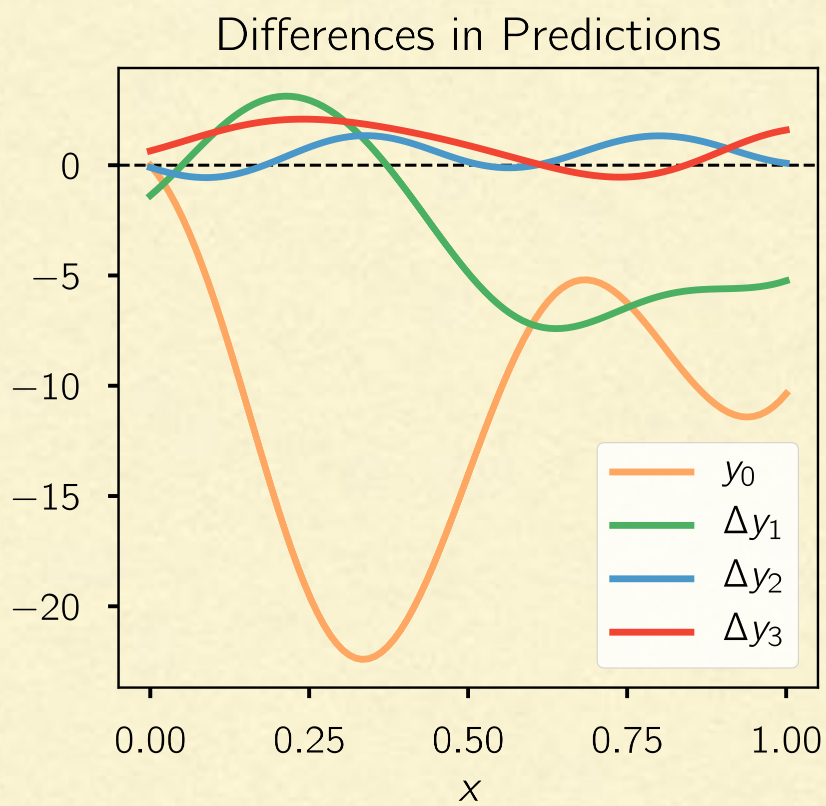

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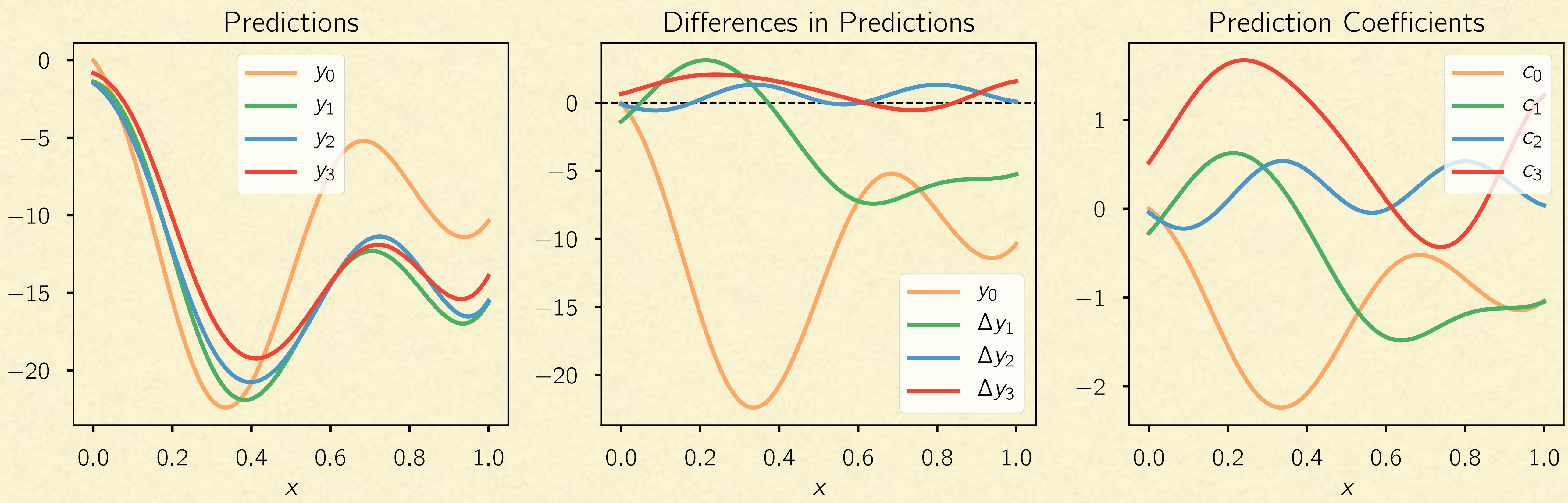

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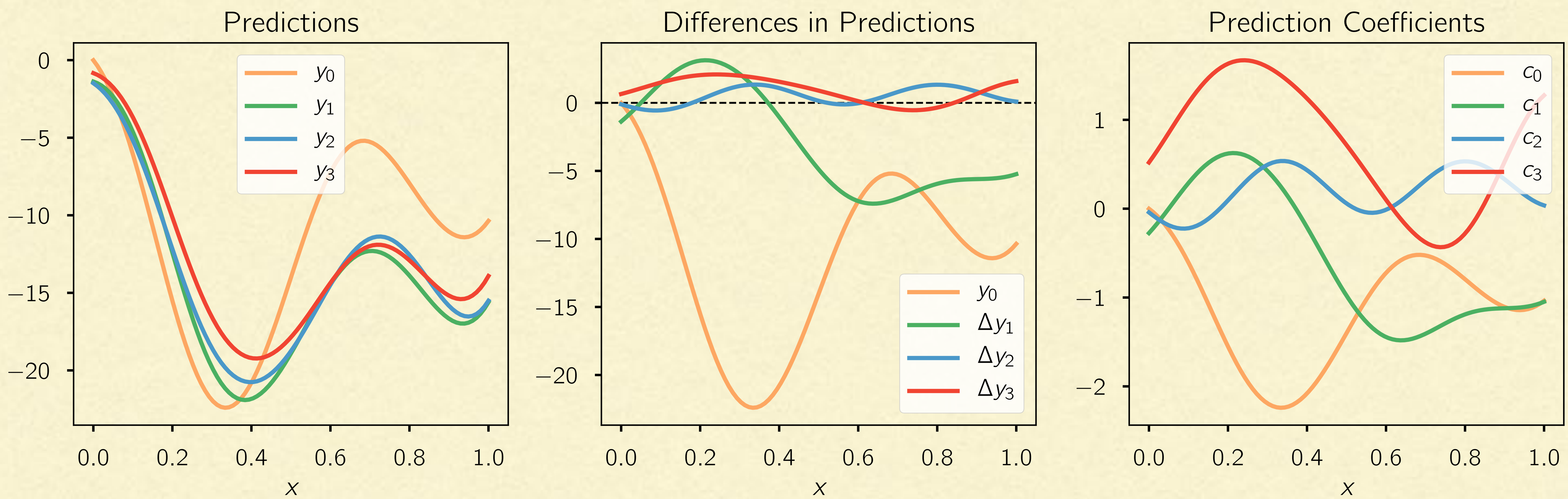

This is what a healthy observable expansion looks like: bounded coefficients, that do not grow or shrink with order.

Discrepancy model

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- $\delta y_{\text{th}} = y_{\text{ref}}(p)[c_{k+1}Q^{k+1} + c_{k+2}Q^{k+2} + \ldots]$
- Predictions for model discrepancy size AND growth with p
- How much do "fine details matter" as we go to higher energy?
- Avoid unintended spurious precision from assumption that model is arbitrarily precise to arbitrarily high energy/short distances

Probability for EFT coefficients

Furnstahl, Klco, DP, Wesolowski, PRC, 2015 after Cacciari and Houdeau, JHEP, 2011

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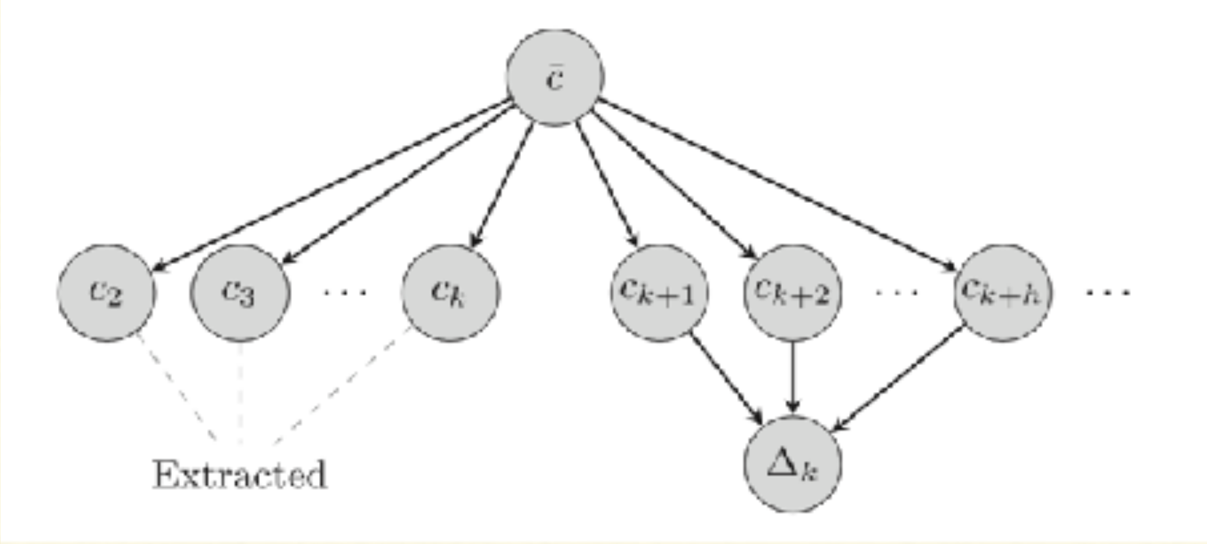
Parameter cbar sets size of all dimensionless coefficients



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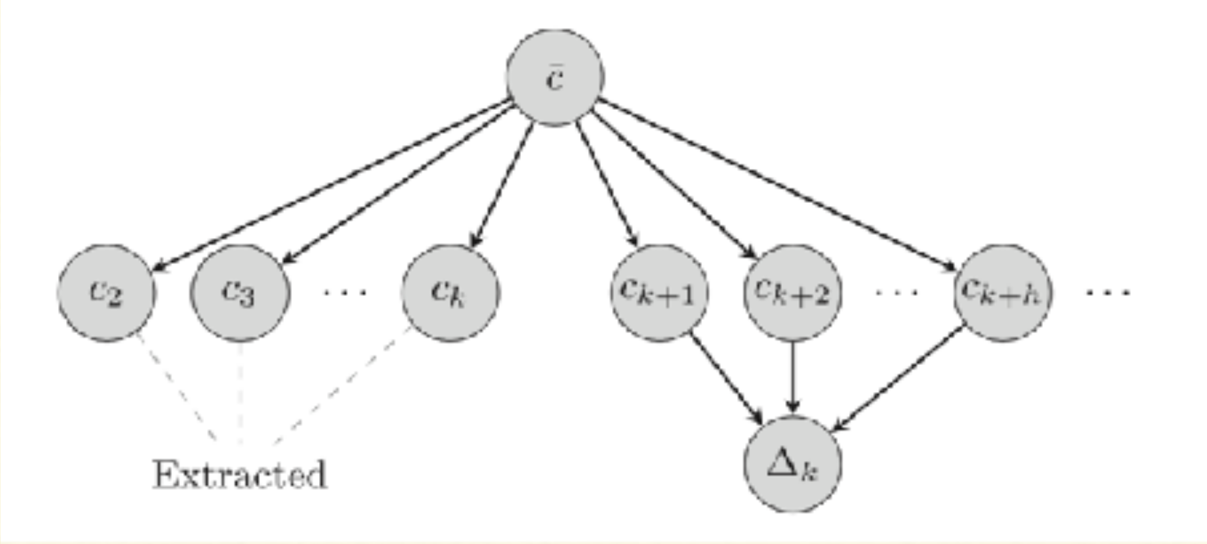
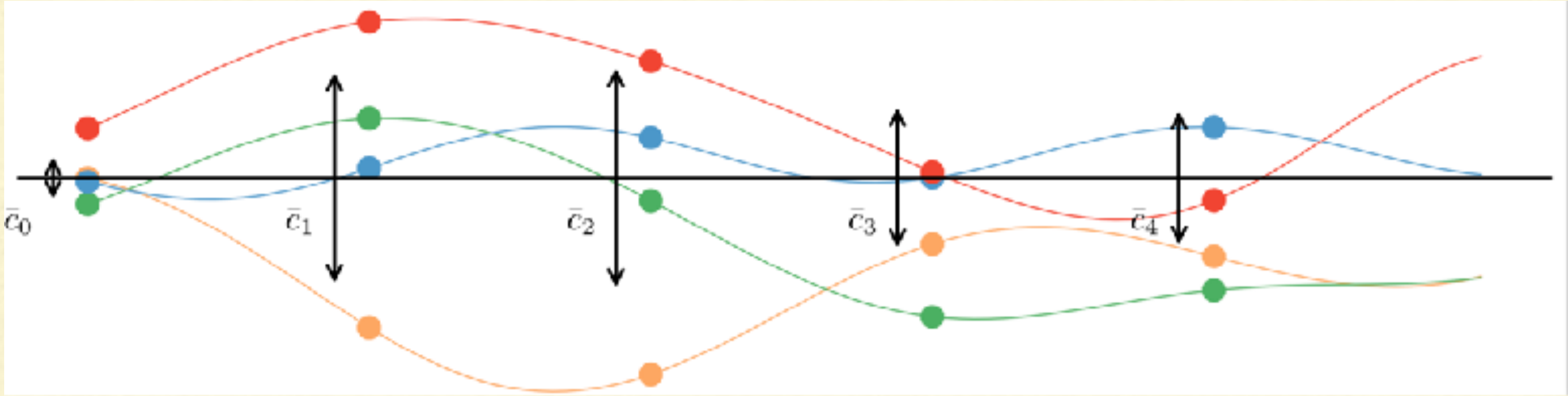




First shot: "uncorrelated model", Errors at different kinematic points are independent, cbar need not be same at different points

Normal naturalness

Furnstahl, Klco, DP, Wesolowski, PRC, 2015; Melendez, Furnstahl, Wesolowski, PRC, 2017

- c_n's are normally distributed, with mean 0 and standard deviation cbar. that is a) fixed or b) distributed uniformly in its logarithm

$$\mathrm{pr}(c_n|\bar{c}) = \frac{1}{\sqrt{2\pi}\bar{c}} e^{-c_n^2/2\bar{c}^2};\ \mathrm{pr}(\bar{c}) \propto \frac{1}{\bar{c}}\theta(\bar{c}-\bar{c}_<)\theta(\bar{c}_> - \bar{c})$$

- Marginalization:

$$\mathrm{pr}(c_{k+1}|c_0, c_1, \ldots, c_k) = \int_0^\infty d\bar{c}\, \mathrm{pr}(c_{k+1}|\bar{c})\mathrm{pr}(\bar{c}|c_0, c_1, \ldots c_k)$$

$$= \int_0^\infty \frac{d\bar{c}}{\bar{c}^{k+3}} \exp\left(-\frac{c_{k+1}^2}{2\bar{c}^2}\right) \exp\left(-\frac{(k+1)\langle c^2\rangle}{2\bar{c}^2}\right)$$

- Student's t-distribution results:

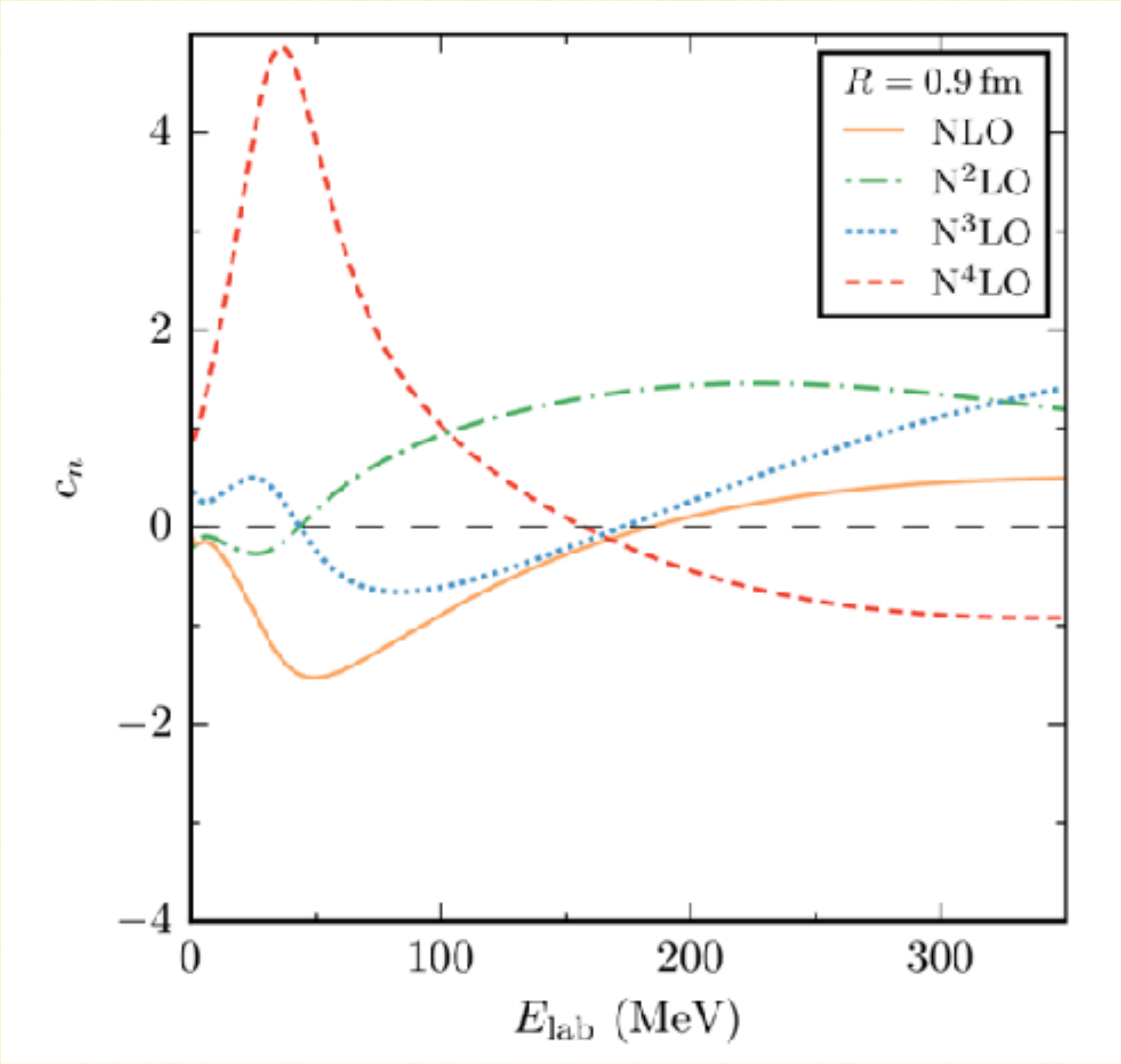
$$\mathrm{pr}(c_{k+1}|c_0, c_1, \ldots, c_k) \propto \frac{\Gamma\left(\frac{k+2}{2}\right)}{\Gamma\left(\frac{k+1}{2}\right)} \left(\frac{(k+1)\langle c^2\rangle}{(k+1)\langle c^2\rangle + c_{k+1}^2}\right)^{(k+2)/2}$$

- DoB intervals computed using known results for this distribution. Size of error bar set by $\langle c^2 \rangle$, k, Q^{k+1}, and y_{ref}.

Error bands for NN observables

Melendez, Furnstahl, Wesolowski, PRC, 2017

EKM SCS, R=0.9 fm potential



Can check error bands for consistency

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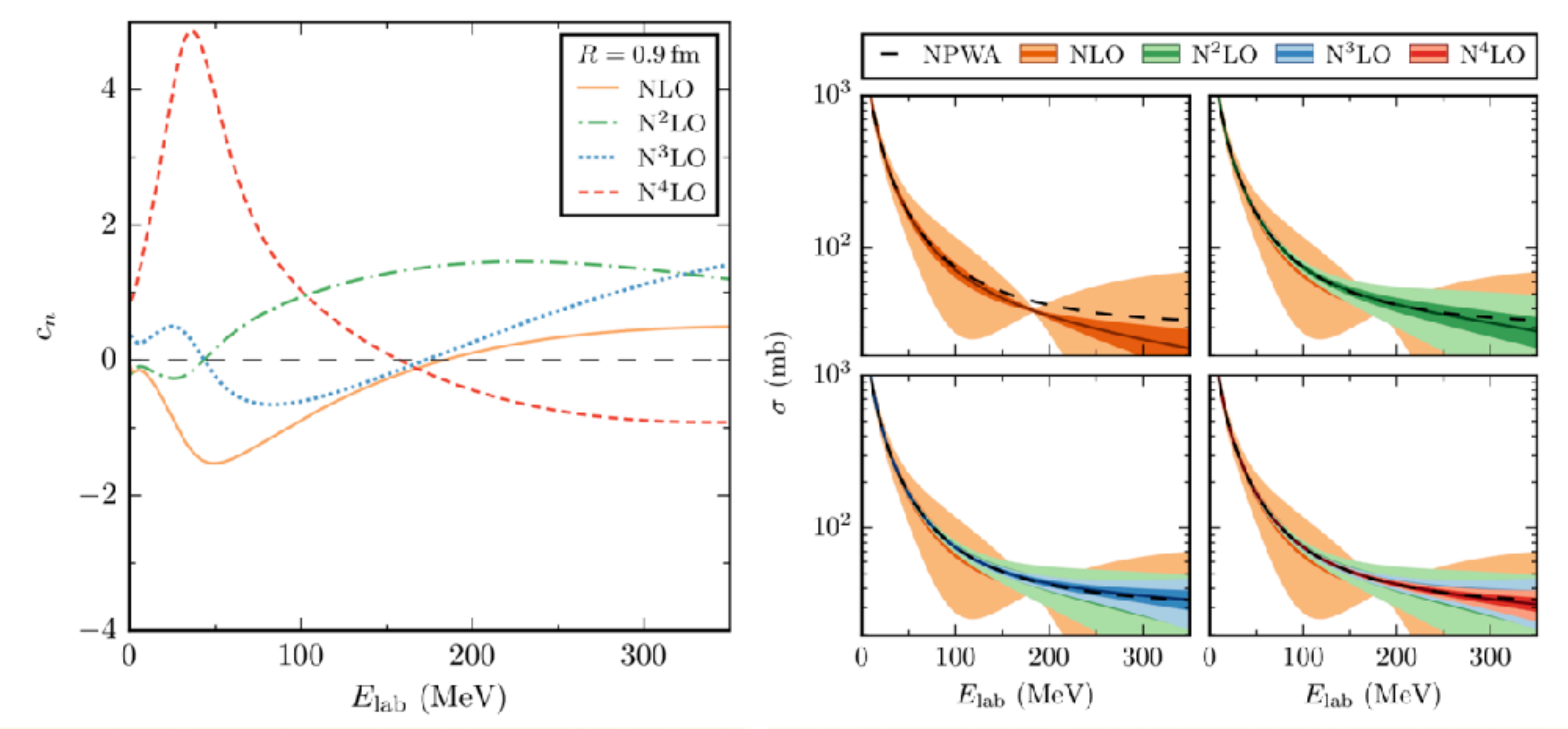

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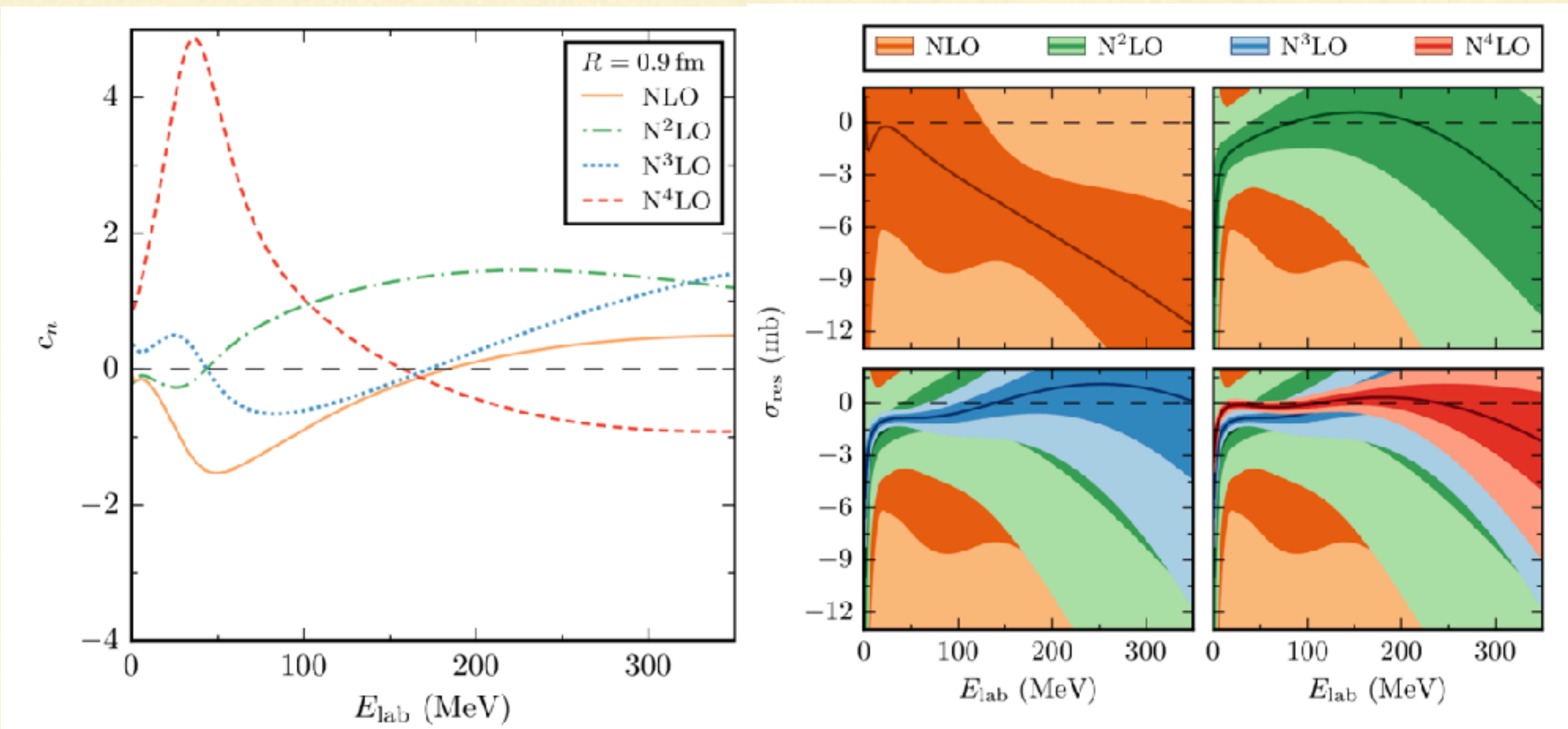

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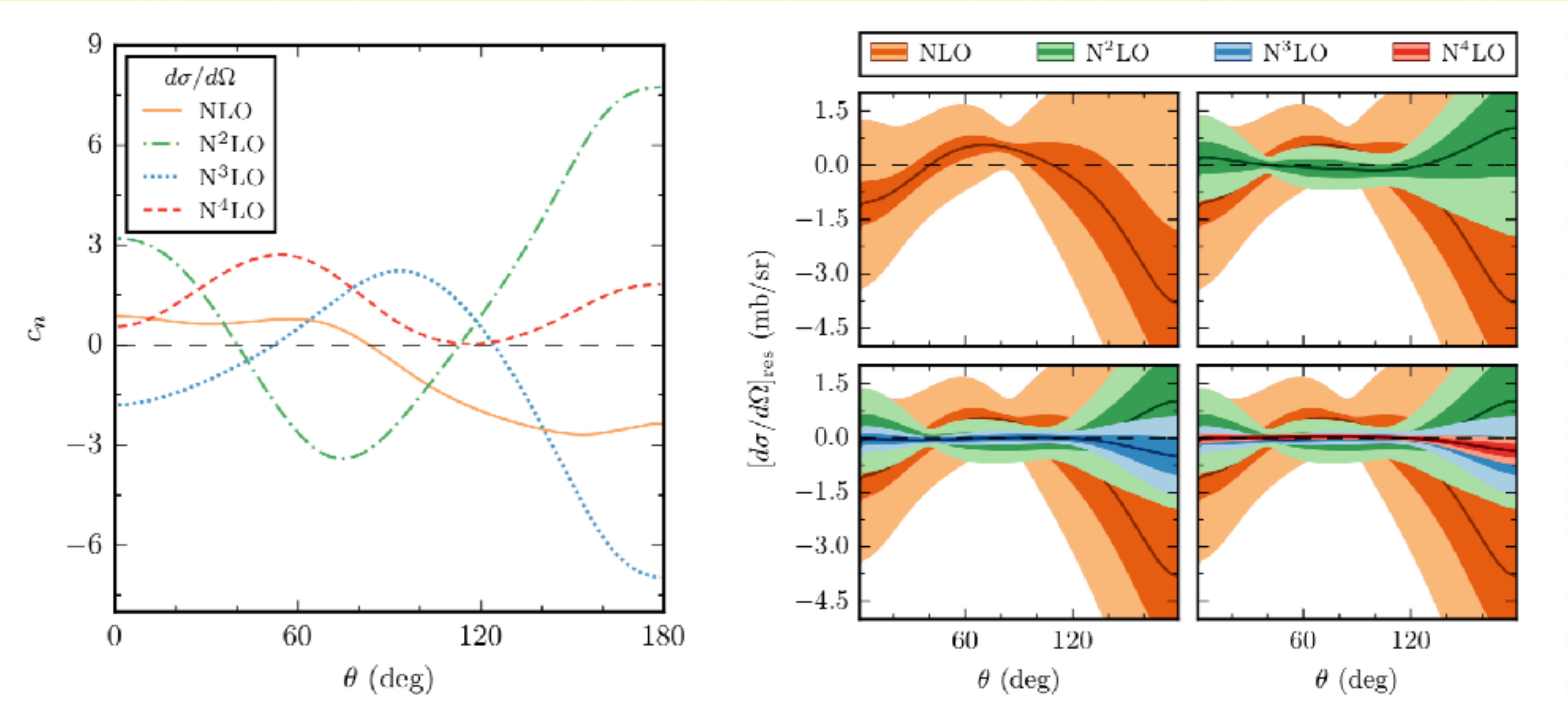
E_{lab}=96 MeV



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Bayesian EFT parameter estimation

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- Marginalization over c's produces revised correlation matrix in standard likelihood, accounts for uncertainties (and correlation structure) induced by omitted terms

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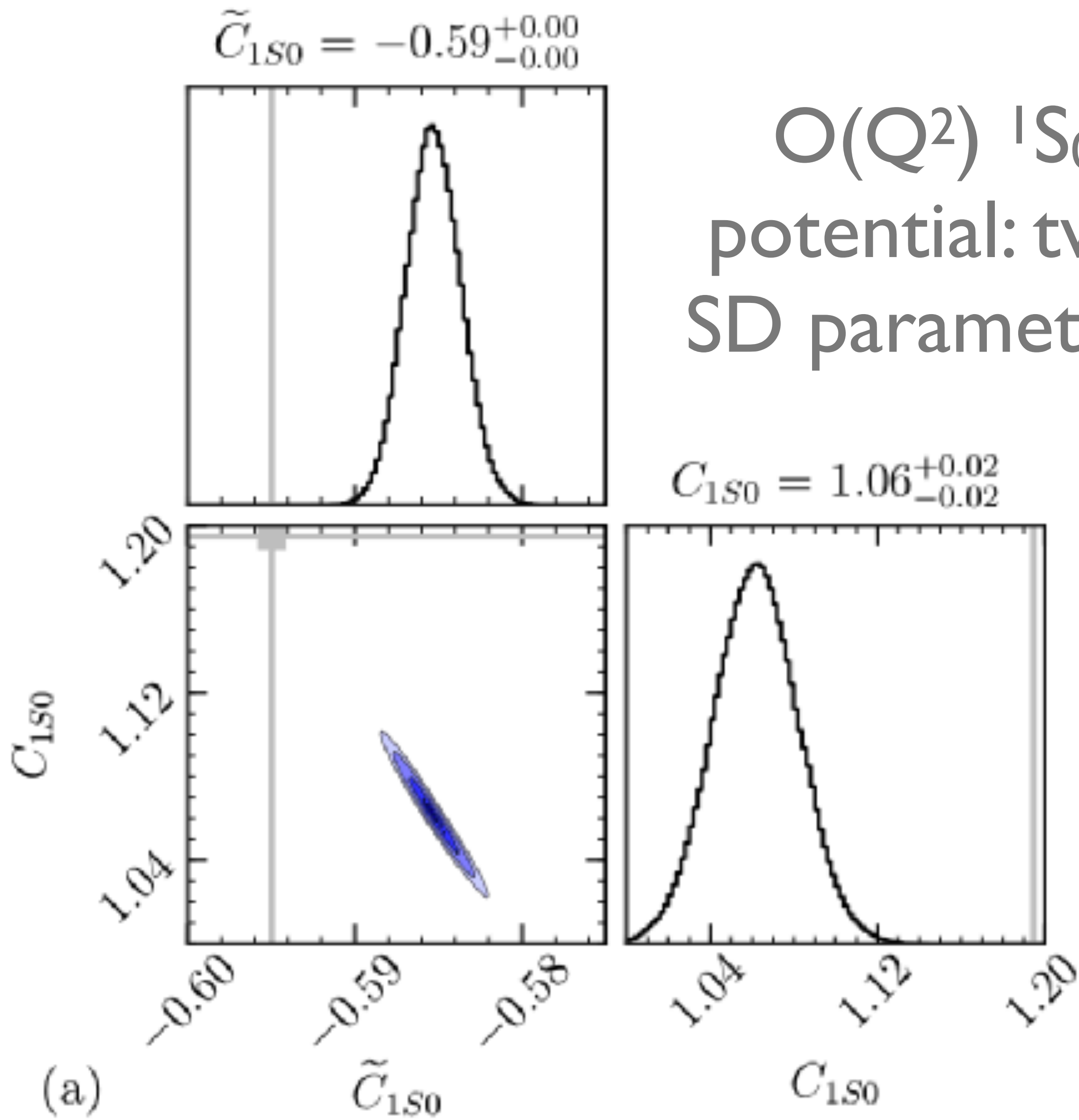
- Normal naturalness (i.e. Gaussian) prior for LECs. Here we take a fixed $\bar{a}$, could also marginalize over it.

Parameter estimates: $^1S_0$

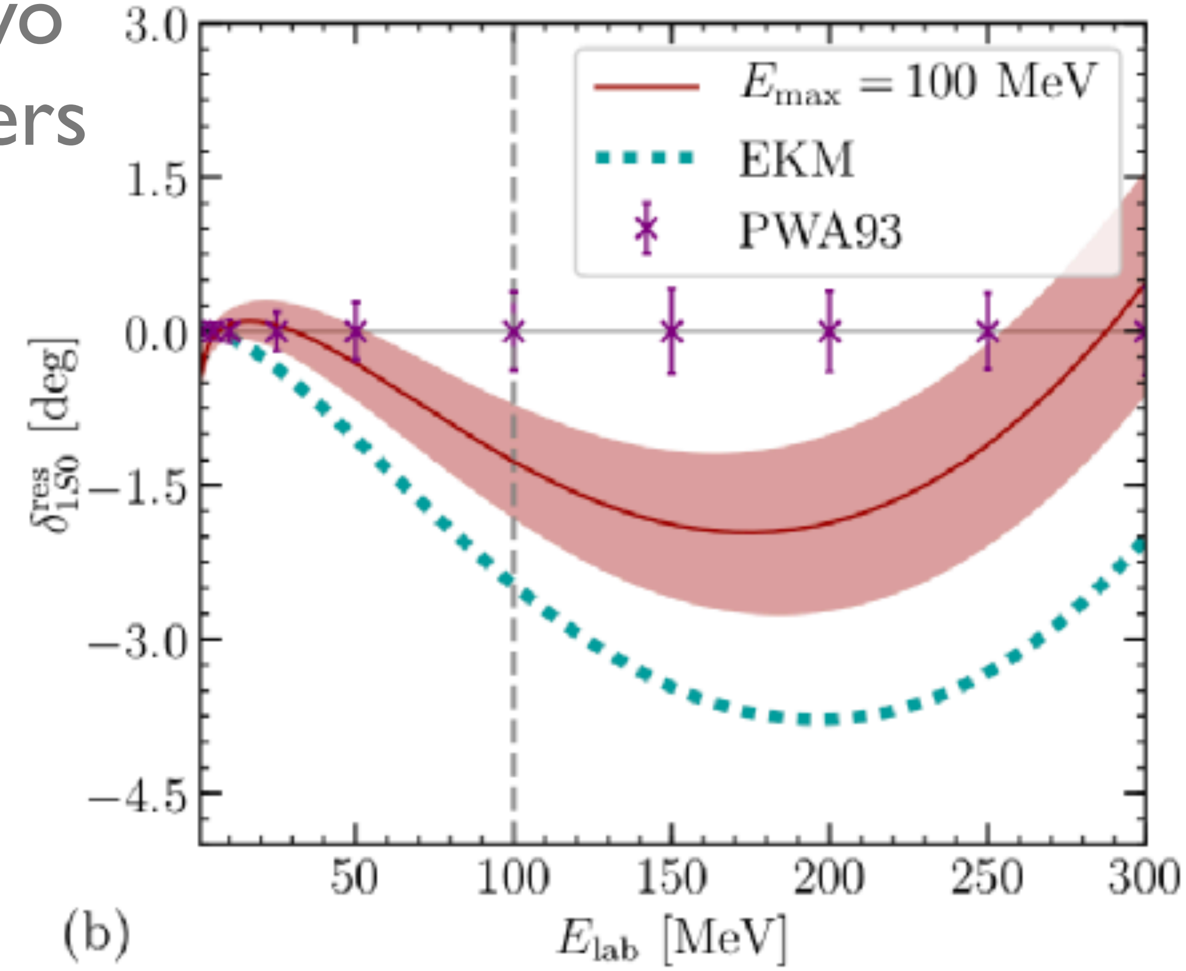
Wesolowski et al., JPG 46, 045102

O(Q^2) $^1S_0$ potential: two SD parameters



EKM SCS potential, R=0.9 fm

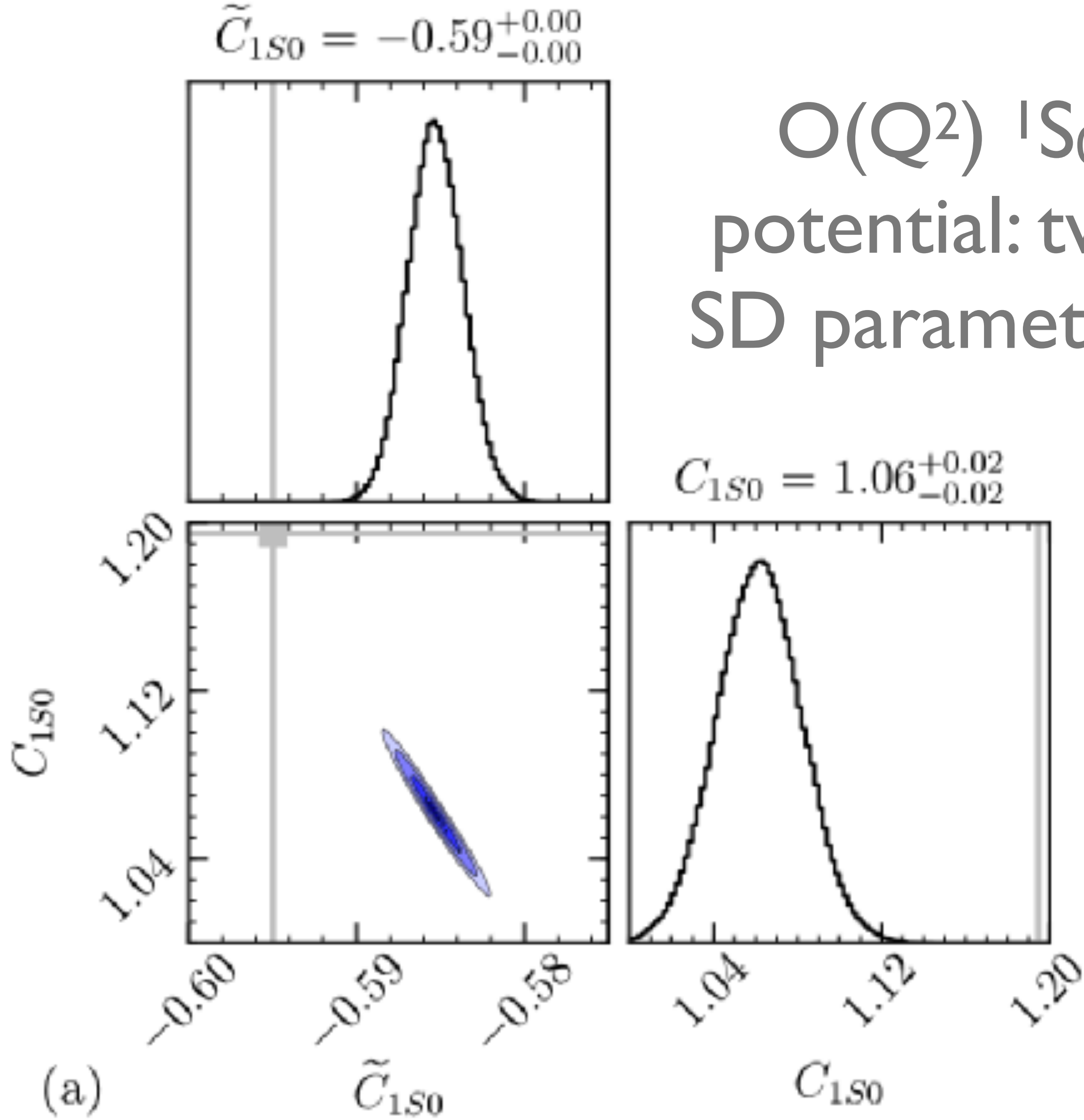
Epelbaum, Krebs, Meißner, EPJA 51, 53 (2015)



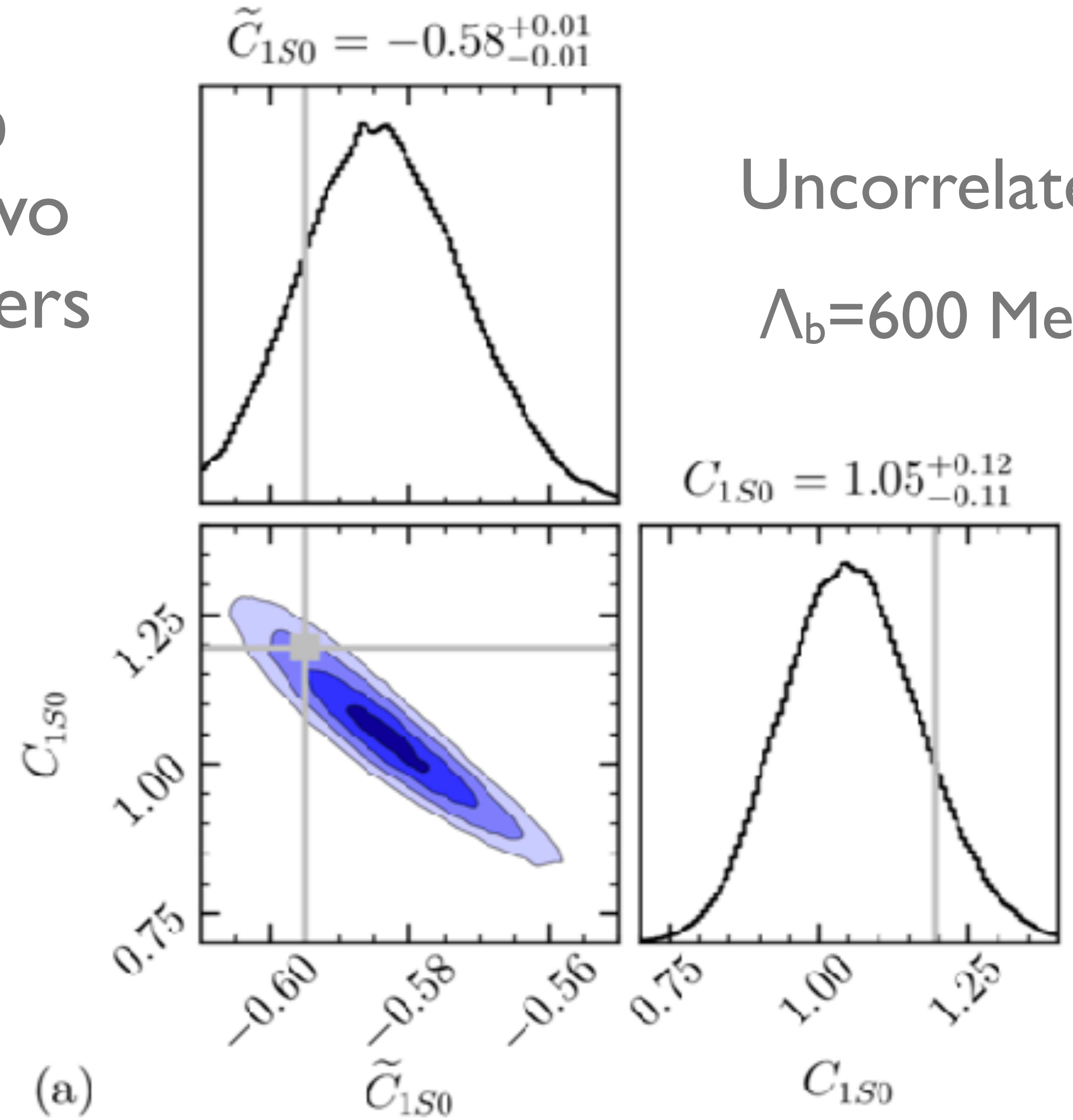
Including truncation errors changes central values and (esp.) errors

Parameter estimates: ${}^1S_0$

Wesolowski et al., JPG 46, 045102



O(Q²) ${}^1S_0$ potential: two SD parameters



Uncorrelated

Λ_b=600 MeV

Including truncation errors changes central values and (esp.) errors

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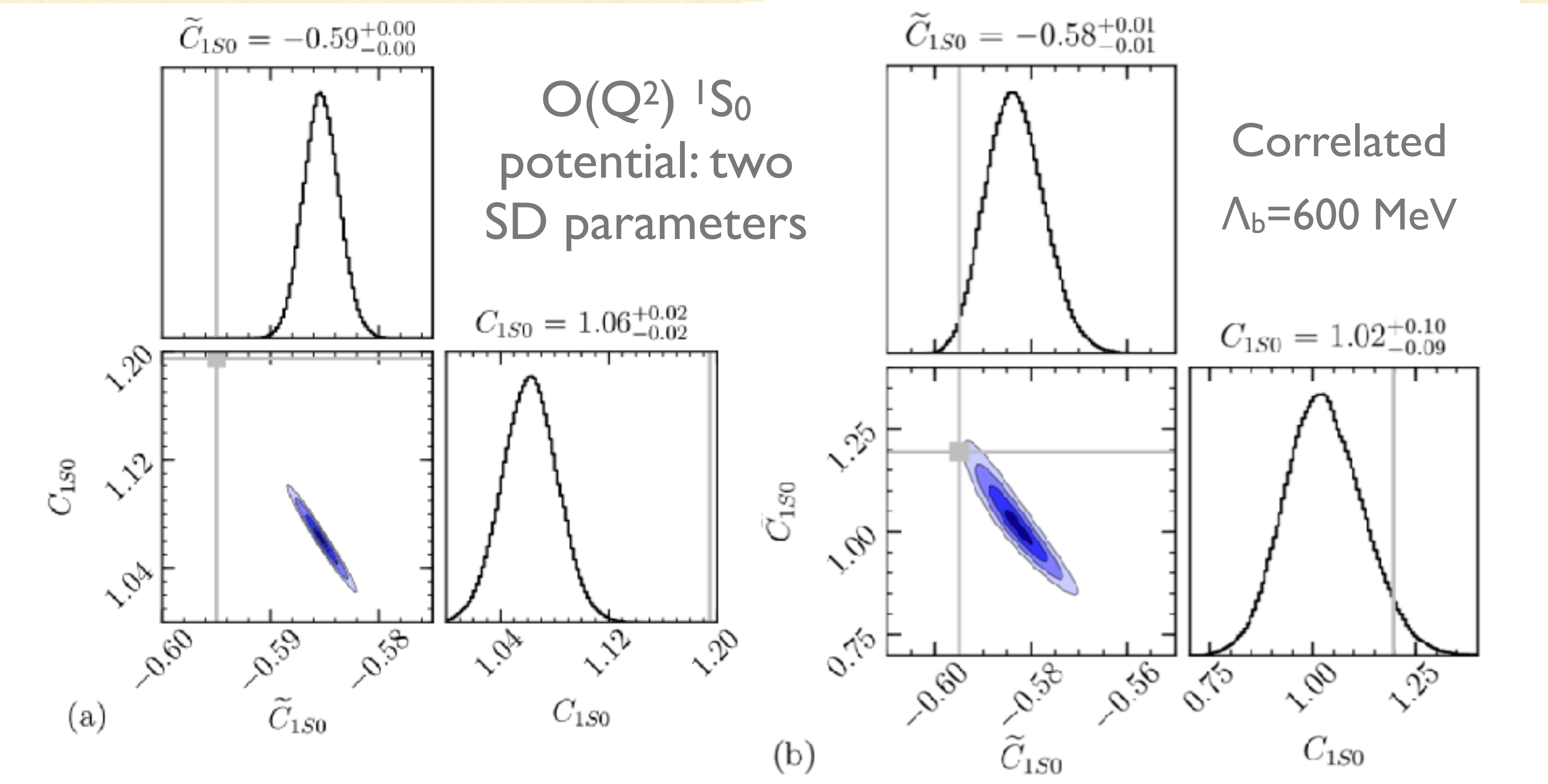

O(Q^2) $^1S_0$ potential: two SD parameters

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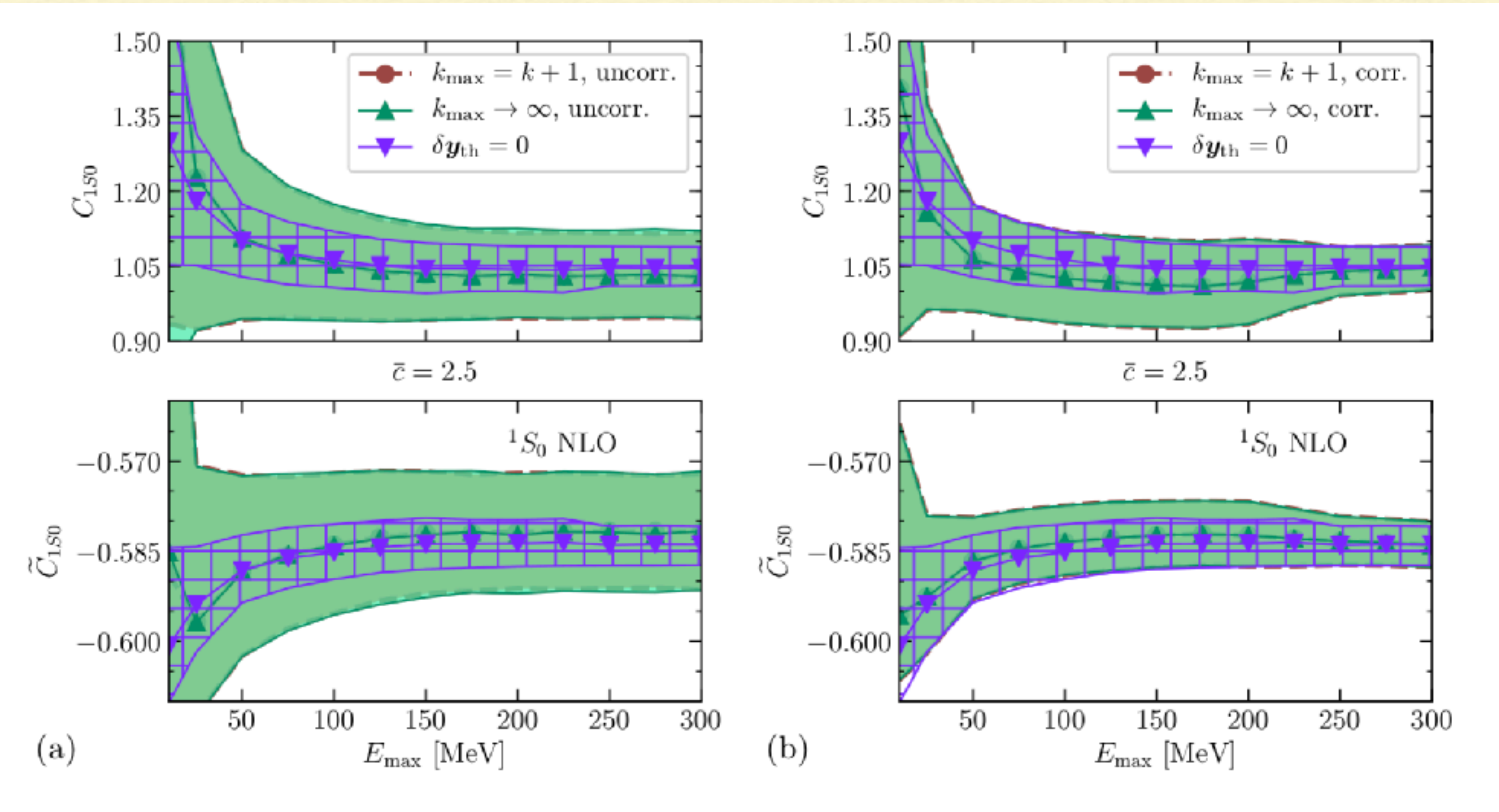
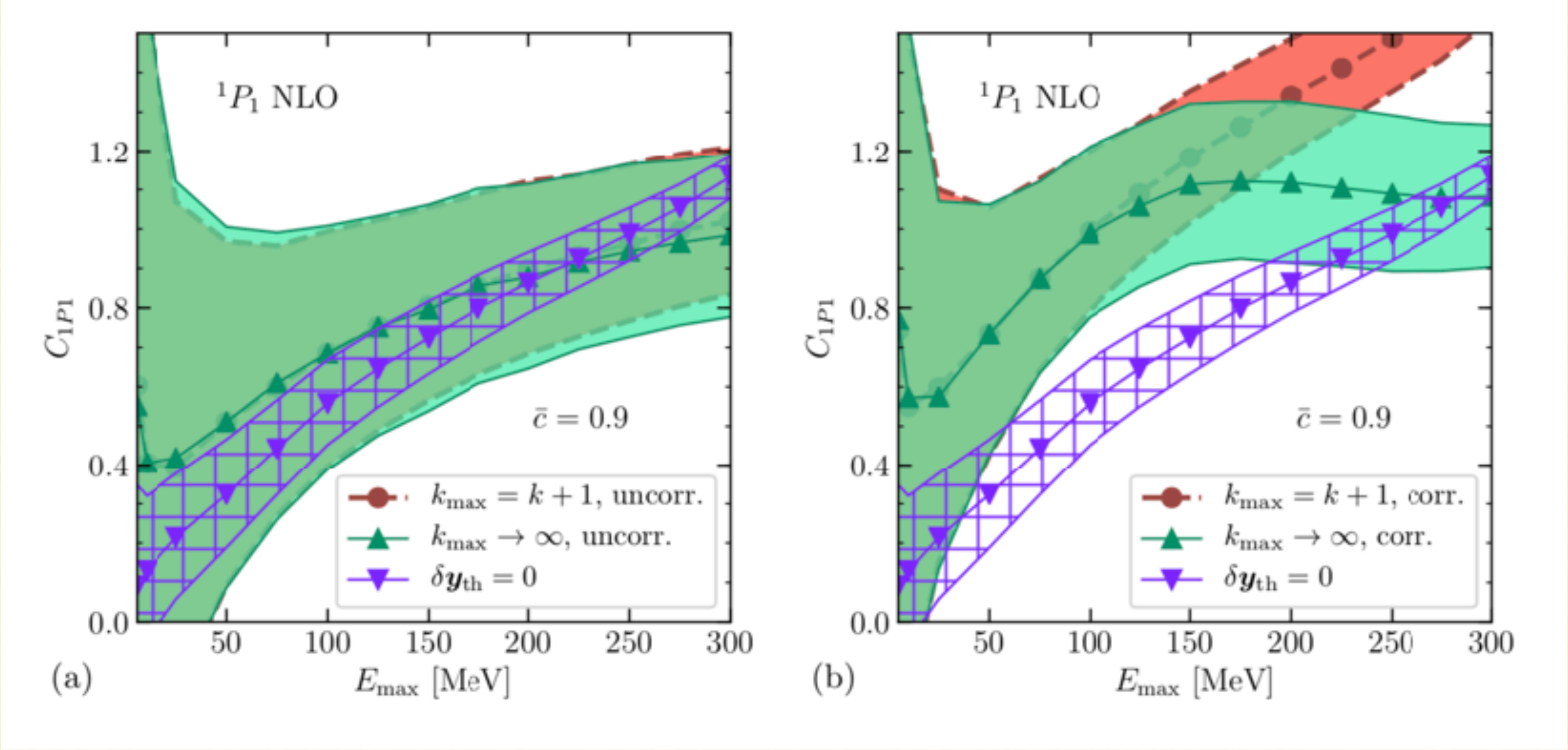
E_{max} plot in the $^1S_0$ at $O(Q^2)$

E_{max} plots: are parameter estimates stable with maximum energy of data?



E_{max} plots in the ${}^1P_1$

Wesolowski et al., JPG 46, 045102



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- Can resum truncation error to all orders (under assumptions about its correlation across orders): tests validity of FOTA

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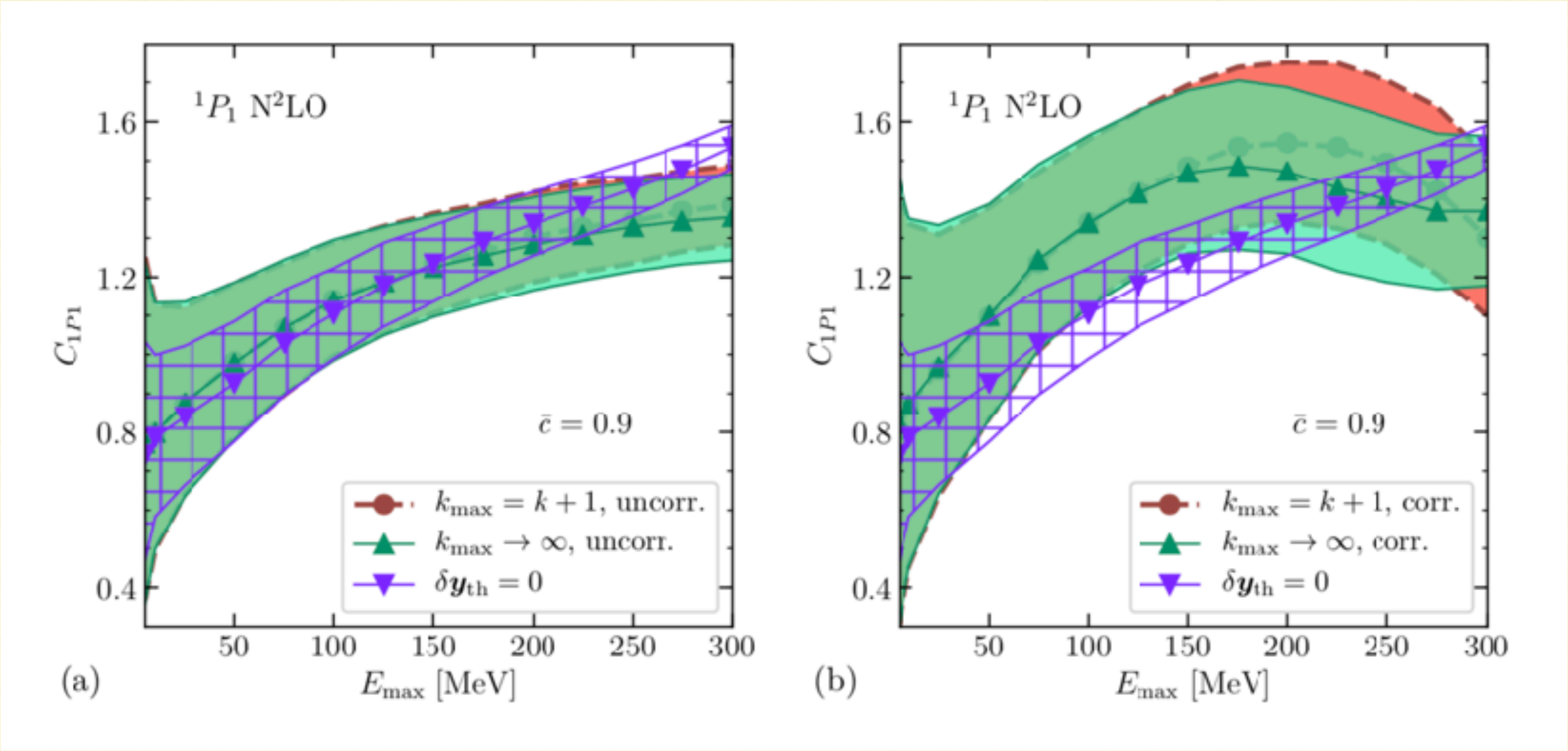

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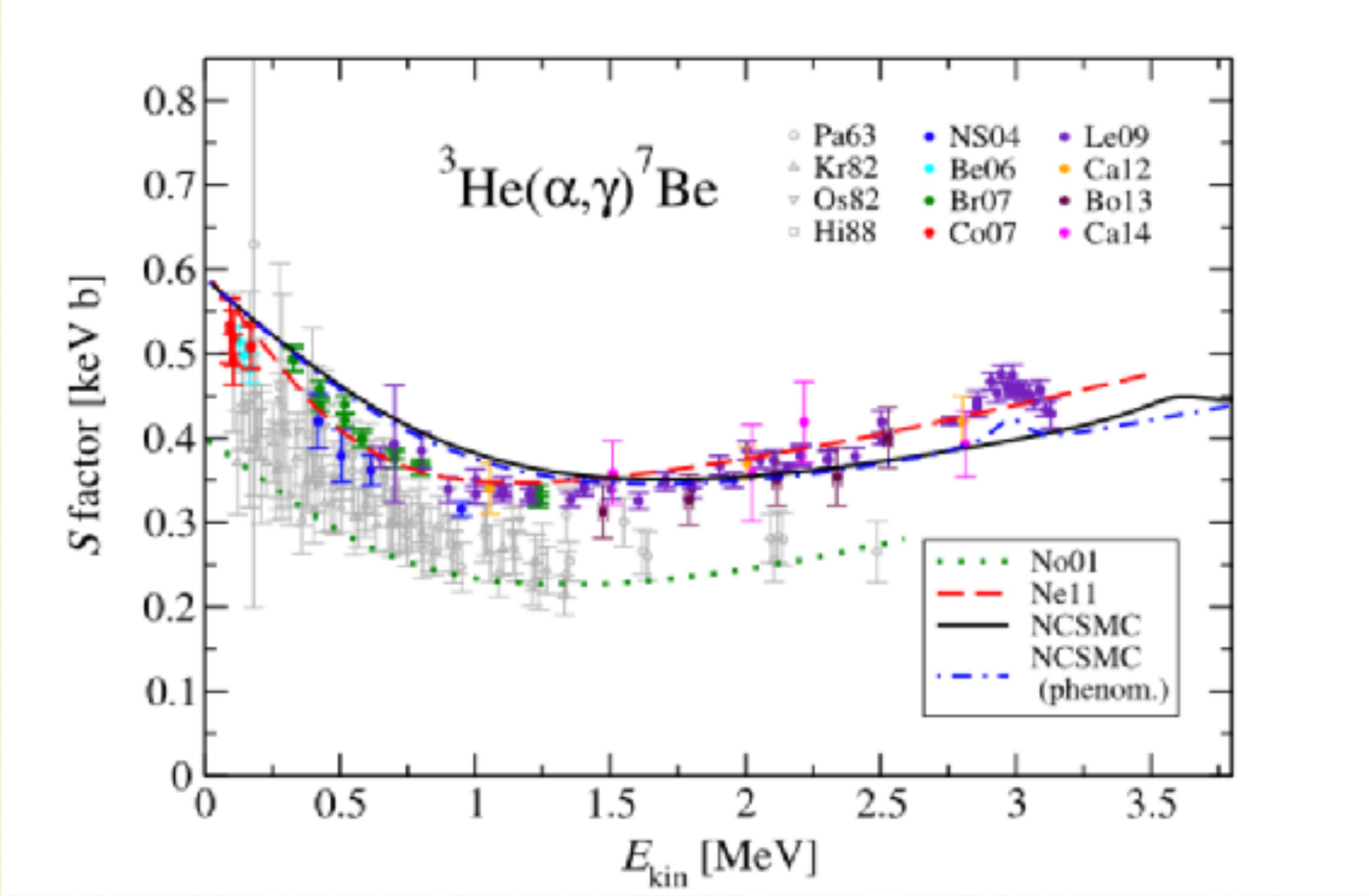
Conclusion

- Effective Field Theory can be used to parameterize and constrain "model uncertainty" through inclusion of truncation errors in analysis
- Bayesian analysis + MCMC sampling can be used to determine EFT parameters
- And with a suitable likelihood the truncation errors' impact on the EFT parameters can be accounted for in the parameter estimation
- Sampling then makes it straightforward to propagate overall uncertainty to desired quantities, e.g., S(0)
- Application to: ^{7}Be(p,γ); ^{3}He(α,γ); ^{3}He(α,α)^{3}He
- Comparison to R-matrix treatment of same reaction very informative

Connecting to *ab initio* calculations



Dohet-Eraly et al., PLB (2016)

- ANC extracted from capture data: $C_{P1/2}^2 + C_{P3/2}^2 = 27 \pm 3$ fm^{-1}
- Significant constraints on s-wave scattering parameters already from capture
- Short-distance parameter L_{E1} is smaller for data and for Nollett's ab-initio based calculation than for cluster models. Pauli principle?