S-factor and scattering parameters from $^{3}\mathrm{He} + {}^{4}\mathrm{He} \rightarrow {}^{7}\mathrm{Be} + \gamma$ data

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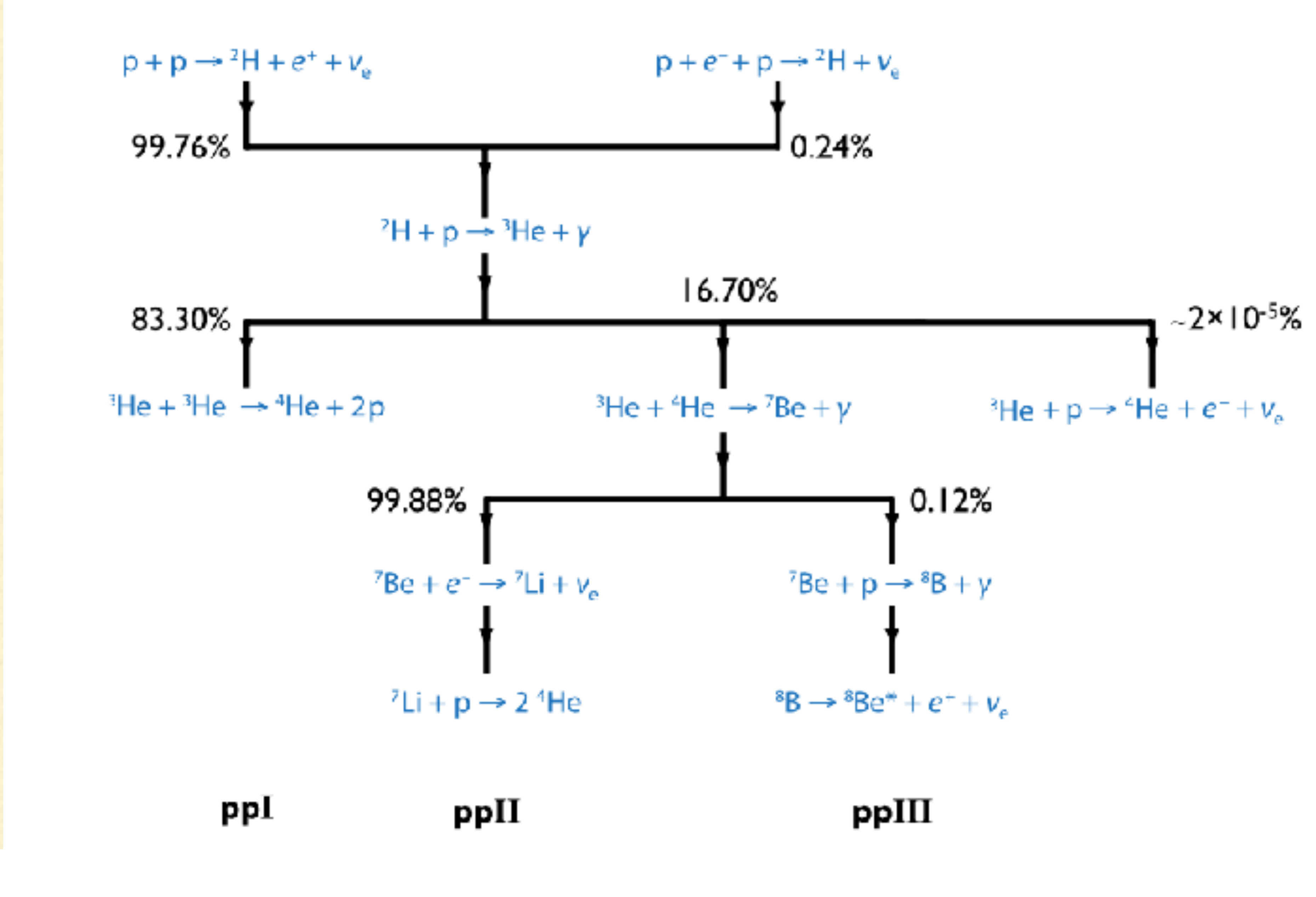
Why is $^3He(^4He,\gamma)$ important?

Adelberger et al., Rev. Mod. Phys. 83, 195 (2011)

- Accurate knowledge of $^3He(^4He,\gamma)$ needed to reliably predict amount of ^7Be in the Sun
- Therefore key for prediction of ^8B solar neutrino flux
- BBN implications, but I will not discuss those here

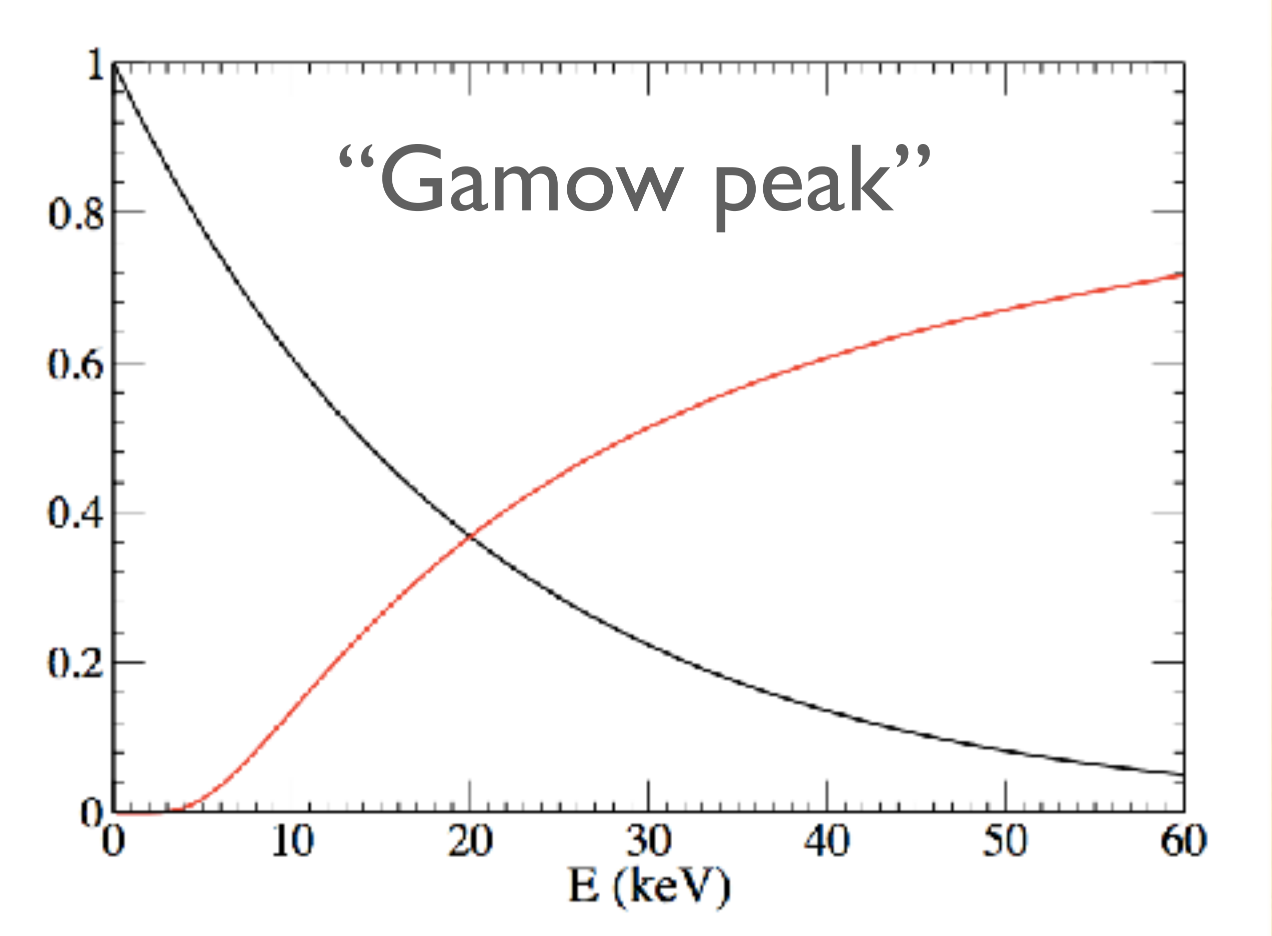


This is an extrapolation problem

Thermonuclear reaction rate $\propto \langle v\sigma \rangle \propto \int_0^\infty dE \, \exp\left(-\frac{E}{k_B T}\right) E \, \sigma(E)$

$$\sigma(E) = \frac{S(E)}{E} \exp\left(-\pi Z_1 Z_2 \alpha_{\rm em} \sqrt{\frac{m_R}{2E}}\right)$$

- E1 capture: $^3He + {}^4He \rightarrow {}^7Be + \gamma$
- Energies of relevance 20 keV

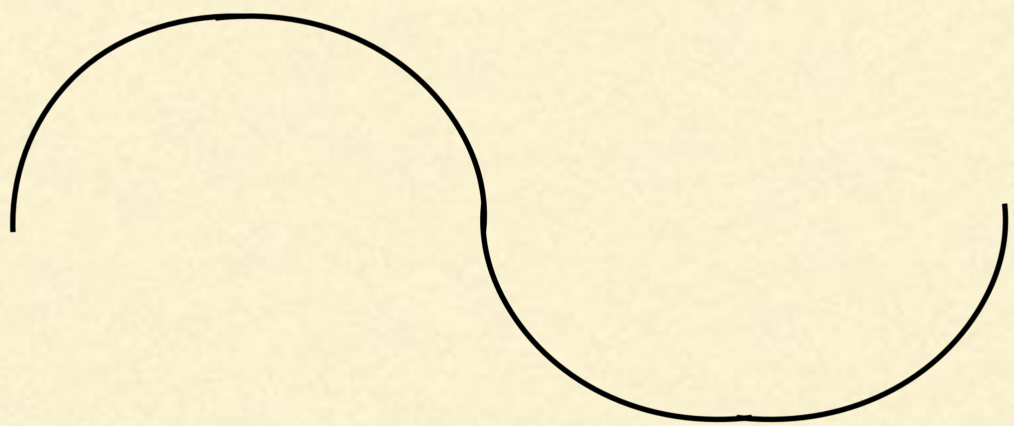


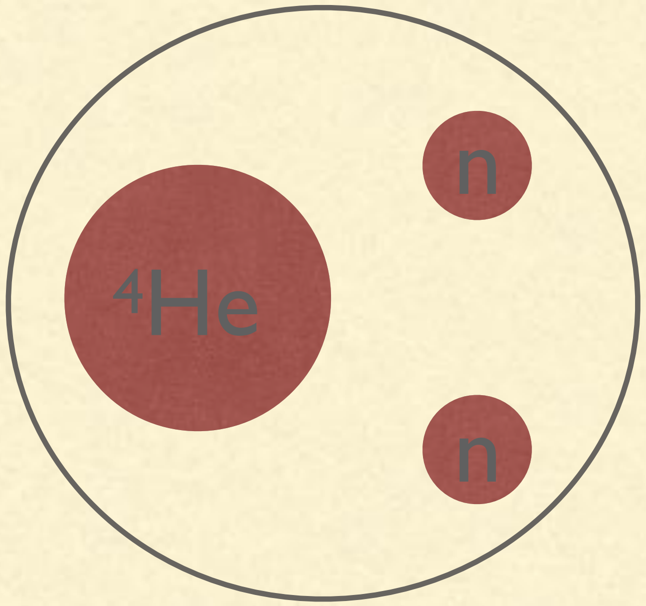
Outline

- $^3He + {}^4He \rightarrow {}^7Be + \gamma$ is an important extrapolation problem
- How Halo Effective Field Theory can help
- Building the pdf for Halo EFT parameters from S-factor and branching-ratio data
- Physics output
- Summary and Future Work

Halo EFT

$\lambda \gg R_{core};\ \lambda \lesssim R_{halo}$





- Define $R_{halo}=\langle r^2\rangle^{1/2}$. Seek EFT expansion in R_{core}/R_{halo}. Valid for $\lambda \lesssim R_{halo}$
- Typically $R \equiv R_{core} \sim 2$ fm. And since $\langle r^2\rangle$ is related to the neutron separation energy we are looking for systems with neutron separation energies less than 1 MeV
- By this definition the deuteron is the lightest halo nucleus, and the pionless EFT for few-nucleon systems is a specific case of halo EFT

Lagrangian: shallow S- and P-states

$$
\begin{aligned}
\mathcal{L} \quad = \quad & c^\dagger \left(i\partial_t + \frac{\nabla^2}{2M}\right) c + n^\dagger \left(i\partial_t + \frac{\nabla^2}{2m}\right) n \\
& + \sigma^\dagger \left[\eta_0 \left(i\partial_t + \frac{\nabla^2}{2M_{nc}}\right) + \Delta_0\right] \sigma + \pi_j^\dagger \left[\eta_1 \left(i\partial_t + \frac{\nabla^2}{2M_{nc}}\right) + \Delta_1\right] \pi_j \\
& - g_0 \left[\sigma n^\dagger c^\dagger + \sigma^\dagger n c\right] - \frac{g_1}{2} \left[\pi_j^\dagger (n \, i\overleftrightarrow{\nabla}_j \, c) + (c^\dagger \, i\overleftrightarrow{\nabla}_j \, n^\dagger) \pi_j\right] \\
& - \frac{g_1}{2} \frac{M-m}{M_{nc}} \left[\pi_j^\dagger \, i\overrightarrow{\nabla}_j \, (nc) - i\overleftrightarrow{\nabla}_j \, (n^\dagger c^\dagger) \pi_j\right] + \dots \, ,
\end{aligned}
$$

- c, n: "core", "neutron" fields. c: boson, n: fermion.
- σ, π_j: S-wave and P-wave fields
- Minimal substitution generates leading EM couplings
- Additional EM couplings at sub-leading order

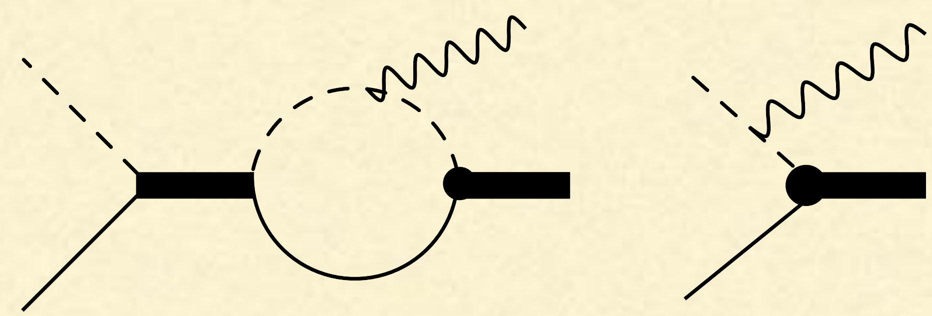
p-wave bound states and capture thereto

Hammer & DP, NPA (2011)

- At LO: p-wave 1n halo described solely by its ANC and binding energy

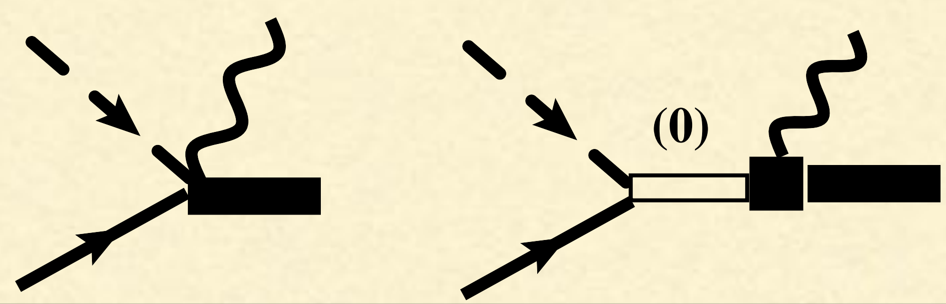
$$u_1(r) = A_1 \exp(-\gamma_1 r)\left(1 + \frac{1}{\gamma_1 r}\right) \qquad \gamma_1 = \sqrt{2m_R B}$$

- Capture to the p-wave state proceeds via the one-body E1 operator: "external direct capture"



$$\mathrm{E1} \propto \int_0^\infty dr\, u_0(r) r u_1(r); \quad u_0(r) = 1 - \frac{r}{a}$$

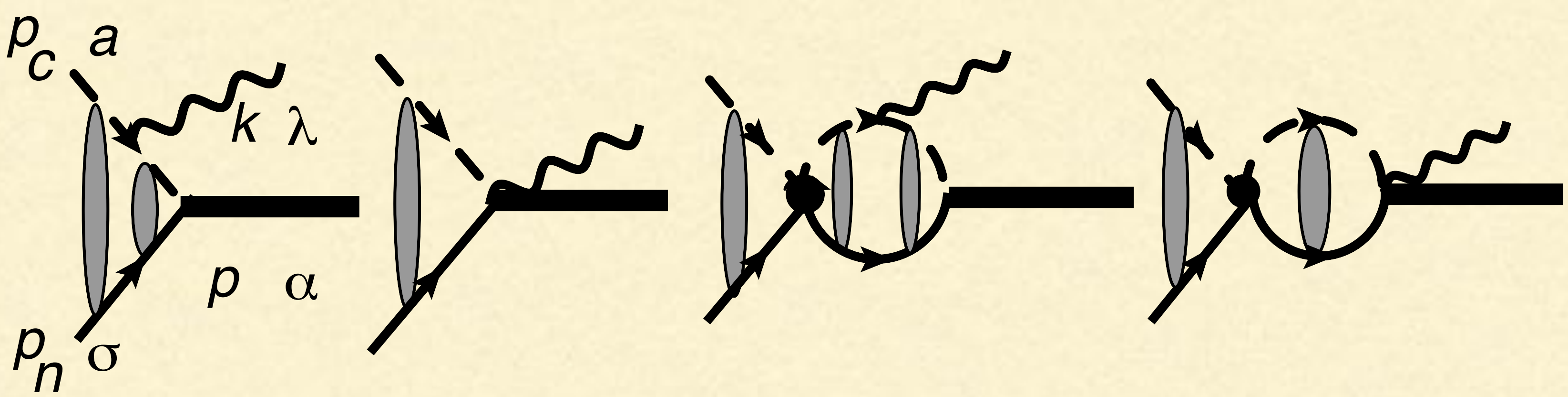
- NLO: piece of the amplitude representing capture at short distances, represented by a contact operator⇒there is an LEC that must be fit



^{3}He + ^{4}He$\rightarrow$^{7}Be + γ_{E1} at LO in Halo EFT

Zhang, Nollett, DP, in preparation; cf. Rupak, Higa, Vaghani, EPJA (2018)

- In this system $R_{core}\sim 1.5$ fm, $R_{halo}\sim 3$ fm; scale of Coulomb interactions: $k_C=Q_c Q_n \alpha_{EM} M_R=17$ MeV; a$\sim$10s of fm, both $\sim R_{halo}$



- Scattering wave functions are linear combinations of Coulomb wave functions F_0 and G_0. Bound state wave function=the appropriate Whittaker function.

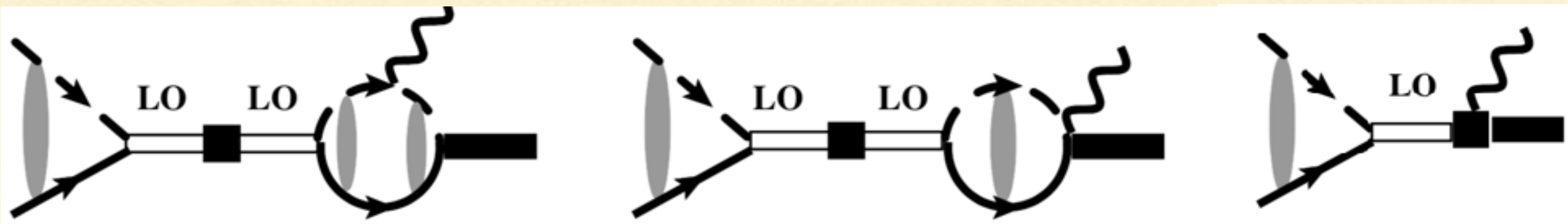
$$S(E) = \frac{e^{2\pi\eta}}{e^{2\pi\eta}-1}\frac{8\pi}{9}(eZ_{eff})^2 k_C \omega^3 C^2 \left[\,|\,\mathcal{S}_{EC}(E;\delta(E)) + |\mathcal{D}(E)|^2\right]$$

Three parameters at leading order

- Can also predict capture to the excited 1/2$^-$ in ^{7}Be

Additional ingredients at NLO

Zhang, Nollett, DP, hys. Lett. B751, 535 (2015), arXiv:1708.04017;
Ryberg, Forssen, Platter, Ann. Phys. (2016)



$$S(E) = \frac{e^{2\pi\eta}}{e^{2\pi\eta}-1}\frac{8\pi}{9}k_C\omega^3 C^2\left[\left|\mathcal{S}_{EC}(E;\delta(E)) + \bar{L}\,\mathcal{S}_{SD}(E;\delta(E))\right|^2 + \left|\mathcal{D}(E)\right|^2\right]$$

Three more parameters at NLO

- Effective range (can add shape parameter which enters at N³LO)
- LECs associated with contact interaction, $\bar{L}$ and $\bar{L}_*$
- Can also consider contact interaction for D-wave capture, $\bar{L}_D$ (enters at N⁴LO)

Data for $^3He + ^4He \rightarrow ^7Be + \gamma_{E1}$

- 59 S-factor data below 2 MeV
  - Seattle (S)
  - Weizman
  - Luna (L)
  - Erna
  - Notre Dame
  - Atomiki
- CMEs
  - 3%
  - 2.2%
  - 2.9%
  - 5%
  - 8%
  - 5.9%
- In general use activation data, to avoid photon emission asymmetry systematic; recoil data from Erna; prompt measurements from Notre Dame
- Deal with CMEs by introducing six additional parameters, ξ_i
- Plus 32 branching-ratio data: CMEs assumed absent there

Building the pdf

- χ^2 needs to include cross-section and branching-ratio data

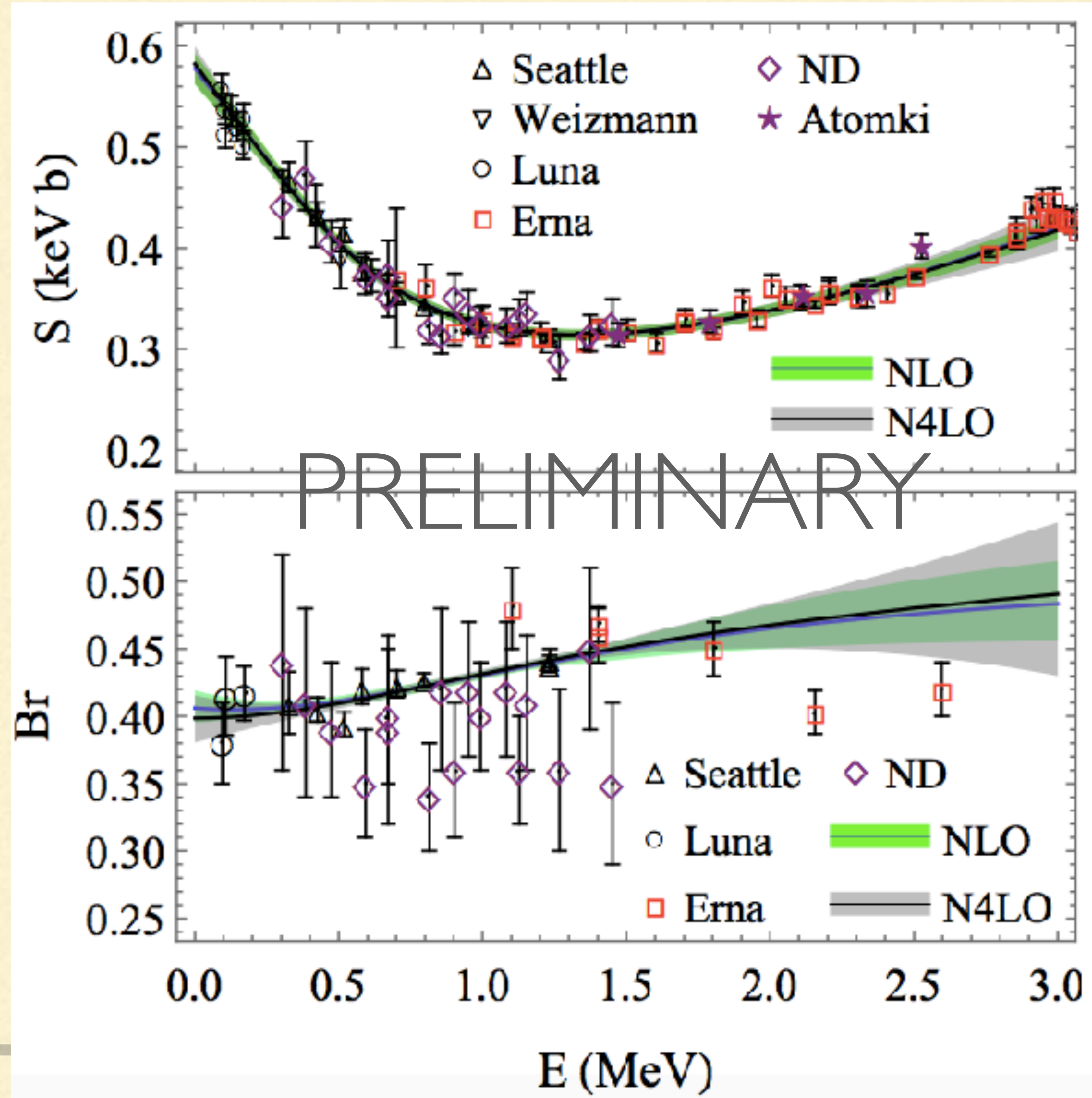
$$\chi^2 \equiv \sum_{J}^{N_{\rm exp}} \left\{ \sum_{j=1}^{N_{s,J}} \frac{\left[(1-\xi_J)S(\overrightarrow{g};E_{Jj}) - D_{Jj}\right]^2}{\sigma_{Jj}^2} + \frac{\xi_J^2}{\sigma_{c,J}^2} \right\} + \sum_{l=1}^{N_{br}} \frac{\left[Br(\overrightarrow{g};E_l) - \tilde{D}_l\right]^2}{\sigma_{br,l}^2}$$

- Mild Bayesian priors:
  - Independent gaussian priors for ξ_i, centered at zero and with width=CME
  - Other EFT parameters, a, r, L, and two ANCs assigned flat priors, corresponding to natural ranges
- Probability $e^{-\chi^2/2}$ sampled using Markov Chain Monte Carlo

^{3}He(^{4}He,γ) results

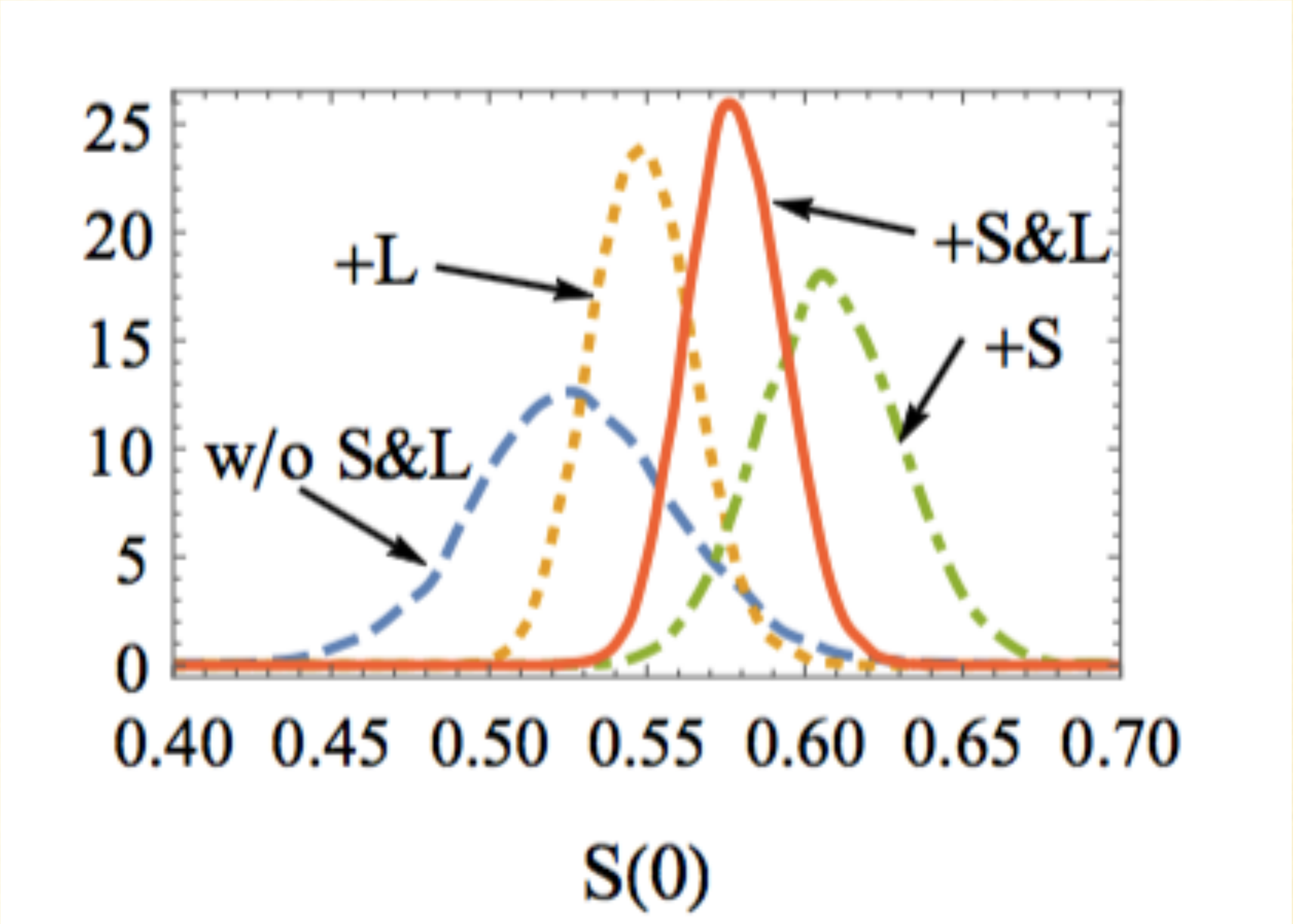
Zhang, Nollett, DP, in preparation cf. Higa, Rupak, Vaghani, EPJA

- E1 external direct capture to a shallow p-wave bound state
- Only one spin channel
- Integral is not dominated by as large r as in ^{7}Be(p,γ)
- More sensitivity to ^{3}He-^{4}He scattering parameterization
- $\chi^2 = 89$
- Bayesian evidence ratio≅4 for NLO cf. N^{4}LO



Impact of different data sets

- Floating data within quoted CME crucial for achieving data consistency
- Pdf gets narrower when either of the precise, low-energy data sets are included
- Seattle data push S(0) to higher values, but still possible to find concordance between Seattle, Luna, and older data



PRELIMINARY

S(0) and its correlants

$$S(0) = 0.578^{+0.015}_{-0.016} \text{ keV b}$$

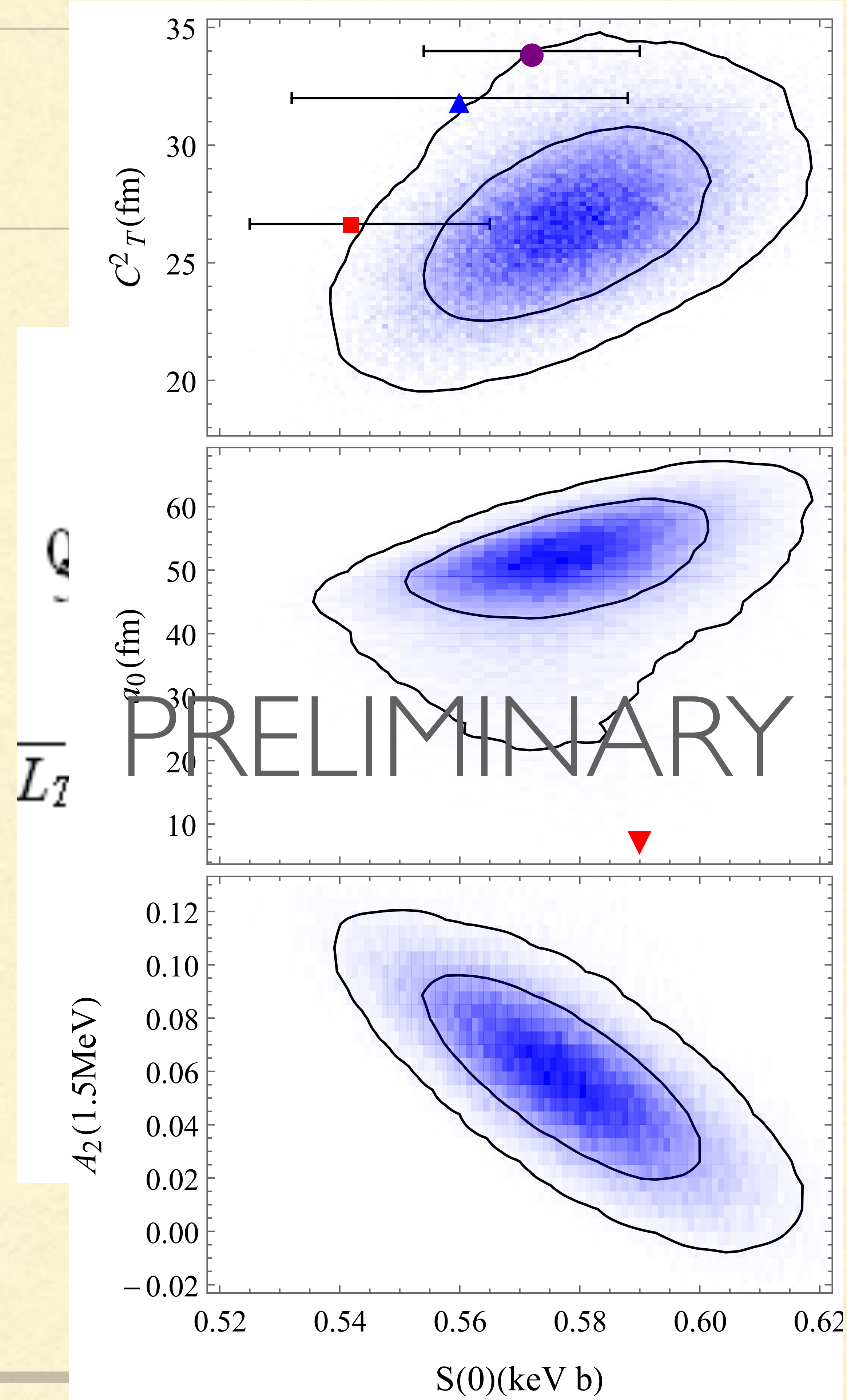
cf. SFII: $S(0) = 0.56 \pm 0.03 \text{ keV b}$

$$Br(0) = 0.406^{+0.013}_{-0.011}$$

Mostly consistent with other analyses, but 1.5σ higher than that of deBoer et al.

How to tell difference?

1. Measure $P_2(\cos\Theta)$ dependence
2. Tight constraints on scattering parameters from capture data alone



Summary

- EFT separates long- and short-distance dynamics, this facilitates reproduction of "reasonable models" through suitable parameter choices
- Extrapolation problem then formulated as a marginalization over models

$$\mathrm{pr}(S(0)|\mathrm{data}, I) = \int d\mathrm{models}\, \mathrm{pr}(S(0)|\mathrm{model}, I)\, \mathrm{pr}(\mathrm{model}|\mathrm{data}, I)$$

- Application of Halo EFT to ^{3}He(^{4}He,γ)^{7}Be produces new S(0), consistent with SFII, but with factor two smaller uncertainty

$$S(0) = 0.578^{+0.015}_{-0.016}\ \mathrm{eV\ b}$$

- This NLO result is a "good fit" and has natural EFT parameters
- Cf deBoer's analysis that produces 1.5σ lower number: illuminates that real data tension is between scattering data and Seattle capture data
- Asymmetry measurements would be illuminating, possible with St. George?

Lessons, future work

- Precise extrapolation can be done even when you don't have lots of data
- Model uncertainty can be accommodated, and standard methods may over-estimate it. It helps to be doing EFT…
- Priors ultimately diagnosable: unconstrained parameters return the prior, and the results we looked at were not sensitive to different choices of prior.
- Projected posterior reveals which combinations of parameters are constrained/affect this observable
- Simultaneous fit to ^{3}He + ^{4}He scattering data is next step
- Especially interesting because of ongoing TRIUMF experiment

References

- Our papers on ^{7}Be + p→^{8}B + γ

Zhang, Nollett, DP Phys. Rev. C 89, 051602 (2014),
Phys. Lett. B751, 535 (2015),
EPJ Web Conf. 113 , 06001 (2016), arXiv:1708.04017

- Other, and more sophisticated, examples of Bayesian Uncertainty Quantification, see BUQEYE collaboration papers
  - Quantifying uncertainties due to omitted higher-order terms

Fursntahl, Klco, DP, Wesolowki, Phys. Rev. C 92, 024005 (2015)
Melendez, Furnstahl, Wesolowski, arXiv:1704.03308

  - Bayesian parameter estimation

Wesolowski, Klco, Furnstahl, DP, Thapaliya, J, Phys,. G 43, 074001 (2016)

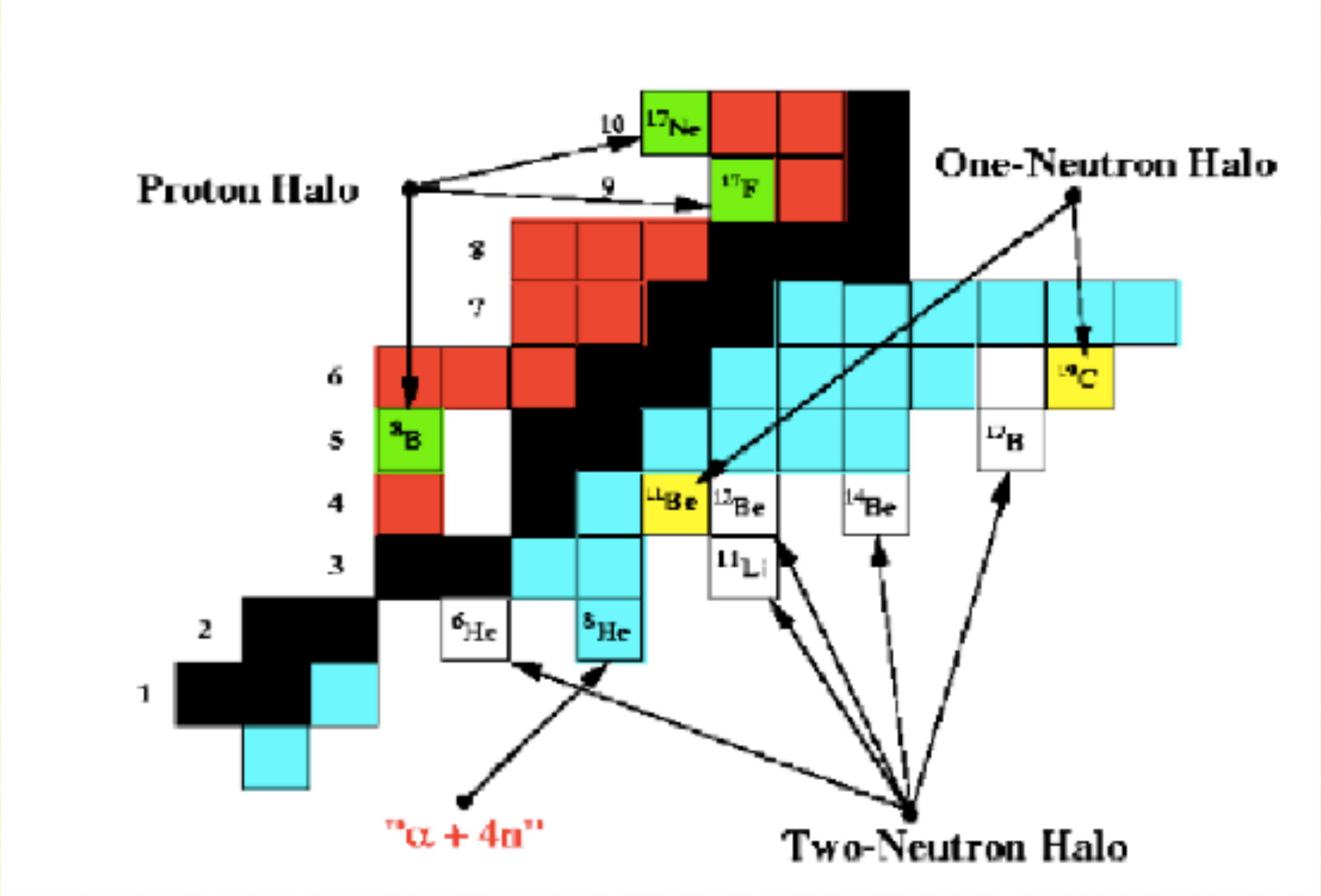
- Review of Halo EFT

Hammer, Ji, DP, J. Phys., G 44, 103002 (2017)

Backup Slides

Halo nuclei

http://nupecc.org



- A halo nucleus as one in which a few (1, 2, 3, 4, ...) nucleons live at a significant distance from a nuclear core.
- Halo nuclei are characterized by small nucleon binding energies, large interaction cross sections, large radii, large E1 transition strengths.

What it does and doesn’t do

It doesn’t:

- Need or discuss spectroscopic factors
- Need or discuss (interior) nodes of the wave function
- Seek to compete with *ab initio* calculations for structure

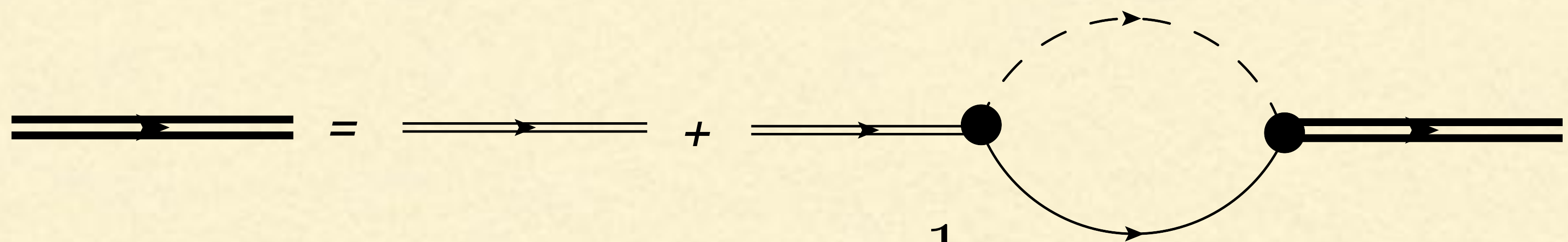
It does:

- Connect structure and reactions, including in multi-nucleon halos
- Collect information from different theories/experiments in one calculation
- Treat same physics as cluster models, in a systematically improvable way
- Provide information on inter-dependencies of low-energy observables, including along the core + n, core + 2n, core + 3n, etc. chain

Dressing the p-wave state

Bertulani, Hammer, van Kolck (2002); Bedaque, Hammer, van Kolck (2003)

- Dyson equation for (cn)-system propagator



$$D_\pi(p) = \frac{1}{\Delta_1 + \eta_1[p_0 - \mathbf{p}^2/(2M_{nc})] - \Sigma_\pi(p)}$$

- Here both Δ_1 and g_1 are mandatory for renormalization at LO

$$\Sigma_\pi(p) = -\frac{m_R g_1^2 k^2}{6\pi}\left[\frac{3}{2}\mu + ik\right]$$

- Reproduces ERE. But here (cf. s waves) cannot take $r_1=0$ at LO

- If $a_1 > 0$ then pole is at $k=i\gamma_1$ with $B_1=\gamma_1^2/(2m_R)$:

$$D_\pi(p) = -\frac{3\pi}{m_R^2 g_1^2}\frac{2}{r_1+3\gamma_1}\frac{i}{p_0 - \mathbf{p}^2/(2M_{nc}) + B_1} + \text{regular}$$

Halo EFT as a "super model"

- Halo EFT is also the EFT of all the models used to extrapolate the cross section in "Solar Fusion II"
- Differences are sub-% level between 0 and 0.5 MeV
- Size of S(0) over-predicted in all models; curves rescaled in SFII fits
- Parameters generally obey $a \sim 1/R_{halo}$, $r \sim R_{core}$, $L \sim R_{core}$, as expected

$C^2_{(^3P_2)}$	$a_{(^3S_1)}$	$r_{(^3S_1)}$
0.200687	15.9977	1.18336
0.200661	24.9966	1.36338
0.200655	33.9933	1.44879
0.109001	-4.14549	6.79899

TABLE IV: The EFT parameters
for scattering length, effective rang