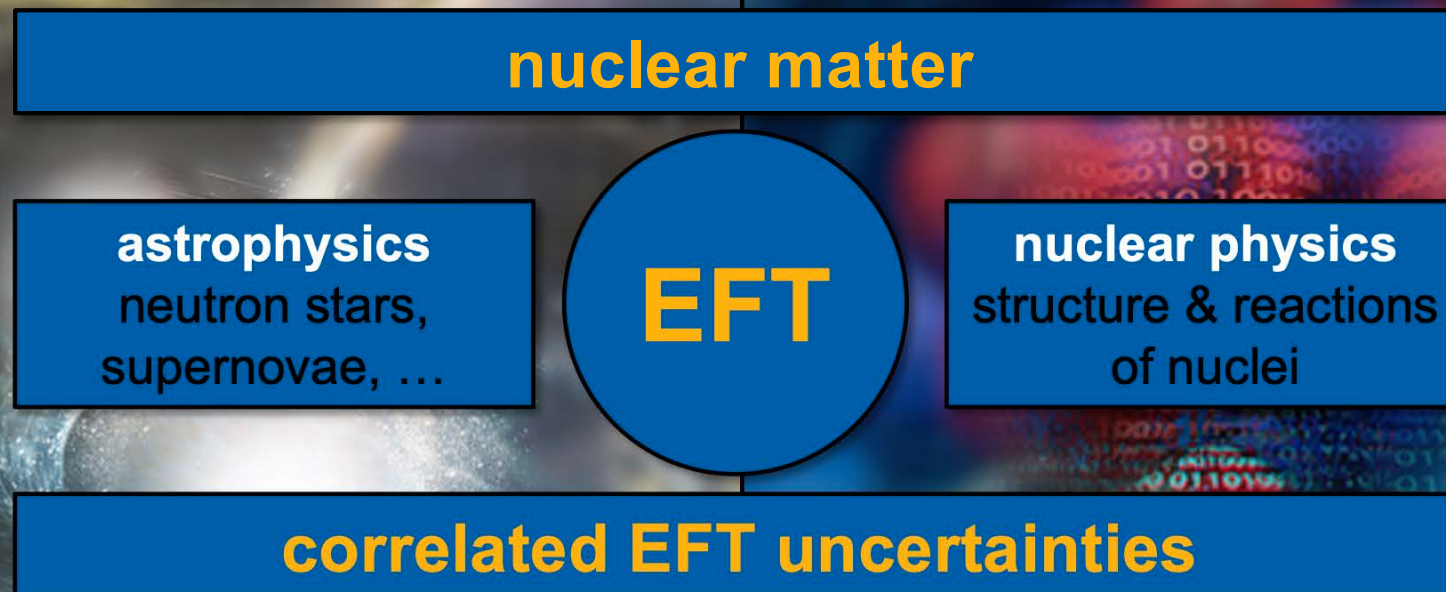


# Quantifying uncertainties in the nuclear-matter equation of state

Berkeley  
UNIVERSITY OF CALIFORNIA

Christian Drischler

Online S@INT | Zoom conference | May 28, 2020



Unterstützt von / Supported by



Alexander von Humboldt  
Stiftung/Foundation

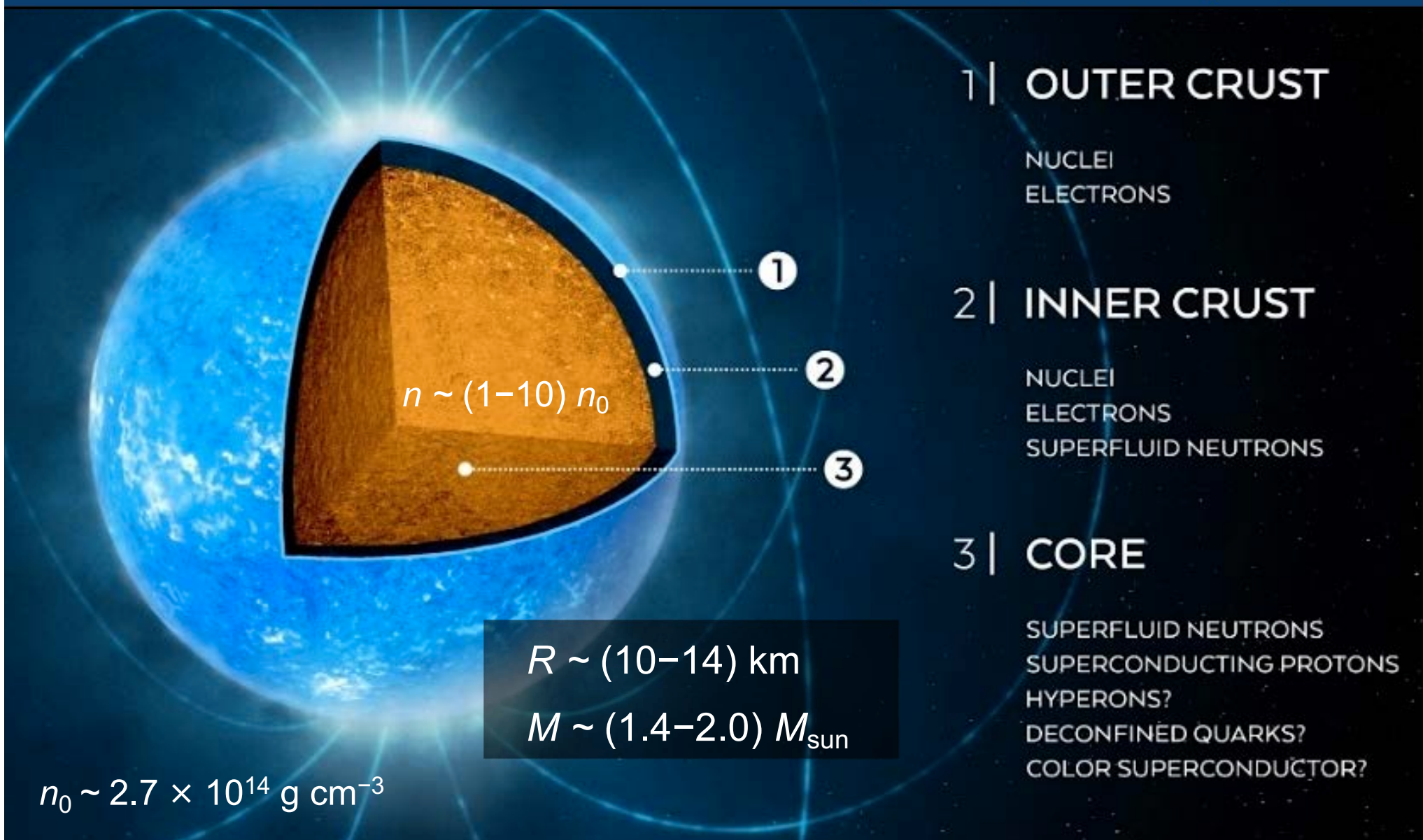


U.S. DEPARTMENT OF  
**ENERGY**



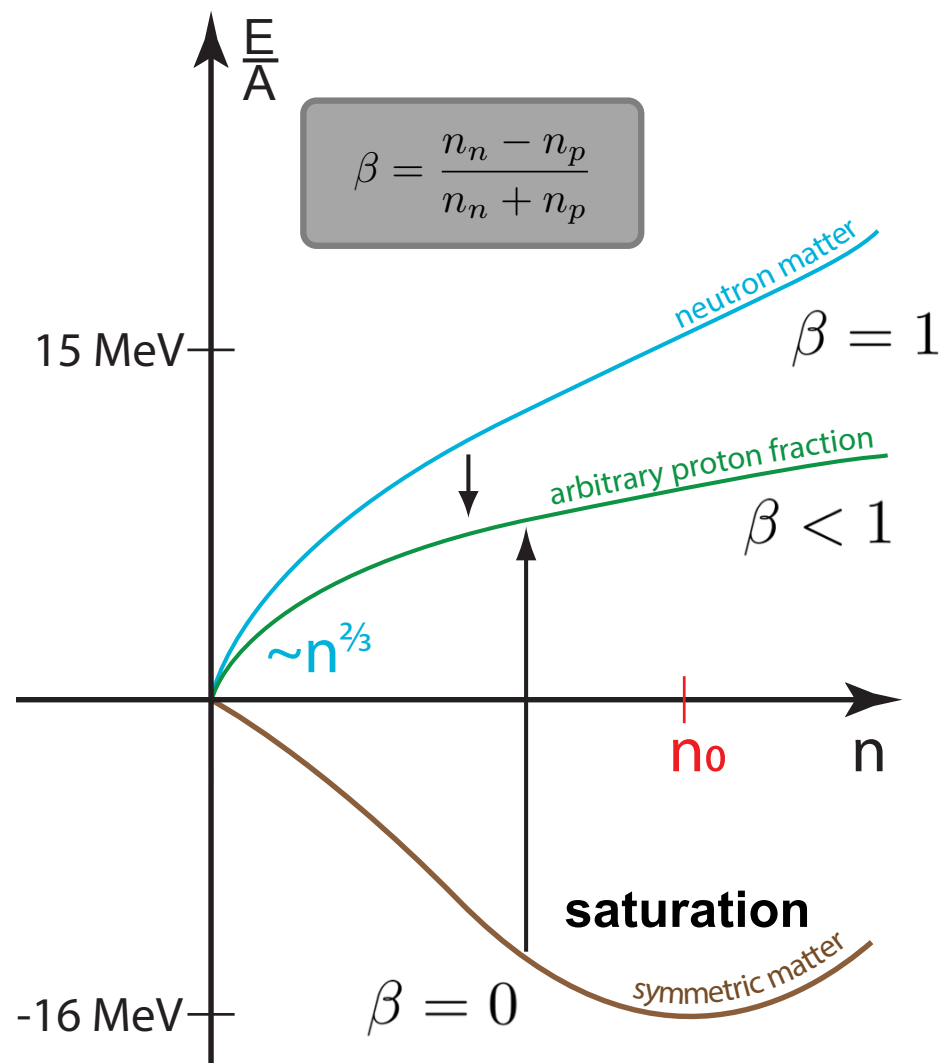
# Quantifying uncertainties in the nuclear-matter equation of state

## Neutron stars



# Quantifying uncertainties in the nuclear-matter equation of state

## Landscape of nuclear matter



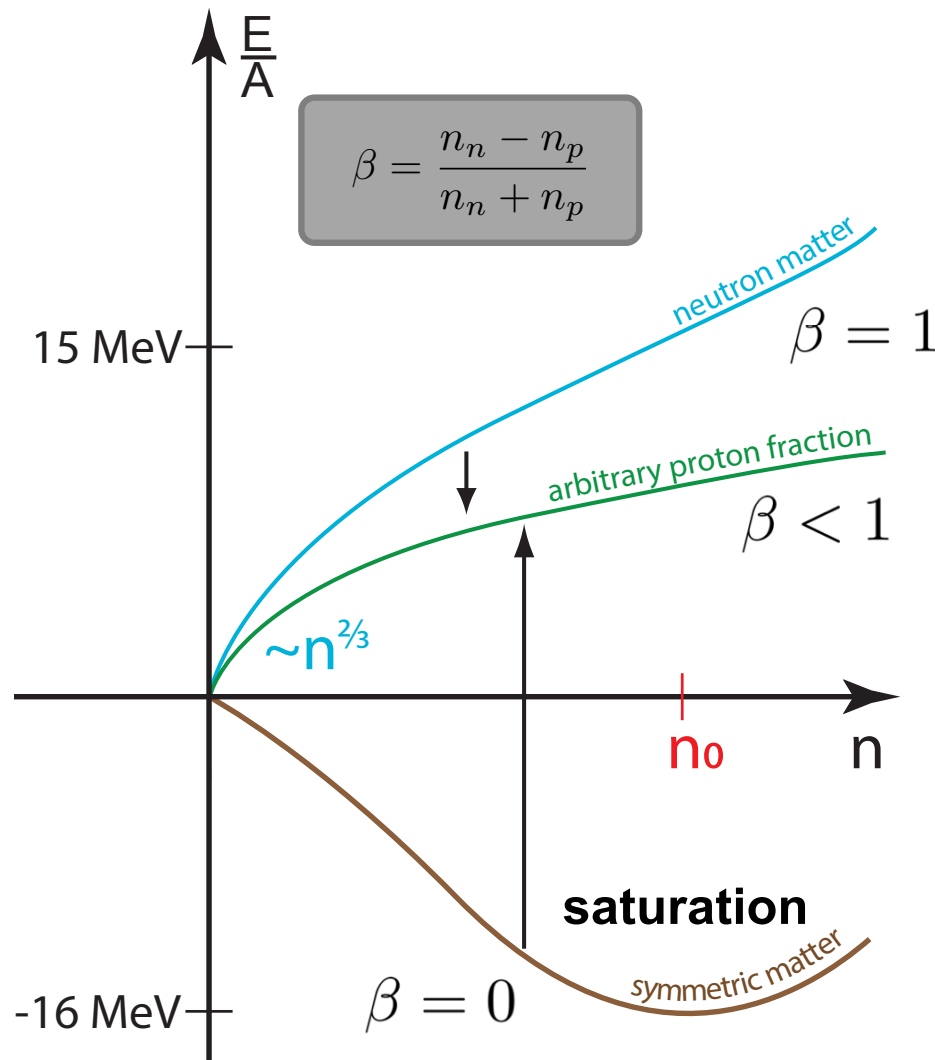
**great progress** in predicting the **EOS** of infinite matter and the structure of **neutron stars** at densities  $\lesssim n_0$

Hebeler, Lattimer *et al.*, APJ **773**, 11  
Carbone, Rios *et al.*, PRC **88**, 044302  
Hagen, Papenbrock *et al.*, PRC **89**, 014319  
Coraggio, Holt *et al.*, PRC **89**, 044321  
Wellenhofer, Holt *et al.*, PRC **89**, 064009  
CD, Carbone *et al.*, PRC **94**, 054307  
Ekström, Hagen *et al.*, PRC **97**, 024332  
Roggero, Mukherjee *et al.*, PRL **112**, 221103  
CD, Hebeler *et al.*, PRL **122**, 042501  
Lonardoni, Tews *et al.*, PRR **2**, 022033(R)  
Piarulli, I. Bombaci *et al.*, PRC **101**, 045801  
...

**But:** existing predictions **only** provided **rough estimates** for the with-density-growing EFT truncation error, without accounting for **correlations**

# Quantifying uncertainties in the nuclear-matter equation of state

## Landscape of nuclear matter



**great progress** in predicting the **EOS** of infinite matter and the structure of **neutron stars** at densities  $\lesssim n_0$

Hebeler, Lattimer *et al.*, APJ **773**, 11

Carbone, Rios *et al.*, PRC **88**, 044302

**needed:** *statistically meaningful comparisons* between nuclear **theory** and recent **observational constraints**

Lonardoni, Tews *et al.* PRR **2**, 022033(R)

Piarulli, I. Bombaci *et al.*, PRC **101**, 045801

...

**But:** existing predictions **only** provided **rough estimates** for the with-density-growing EFT truncation error, without accounting for **correlations**

# Quantifying uncertainties in the nuclear-matter equation of state

Direct detection of gravitational waves

Berkeley  
UNIVERSITY OF CALIFORNIA

[ligo.caltech.edu](http://ligo.caltech.edu)

LIGO

Livingston

Hanford

multi-messenger  
astronomy

+ Virgo  
+ GEO600  
+ ...

Binary Neutron-Star Merger

NSF'S  
10 BIG IDEAS



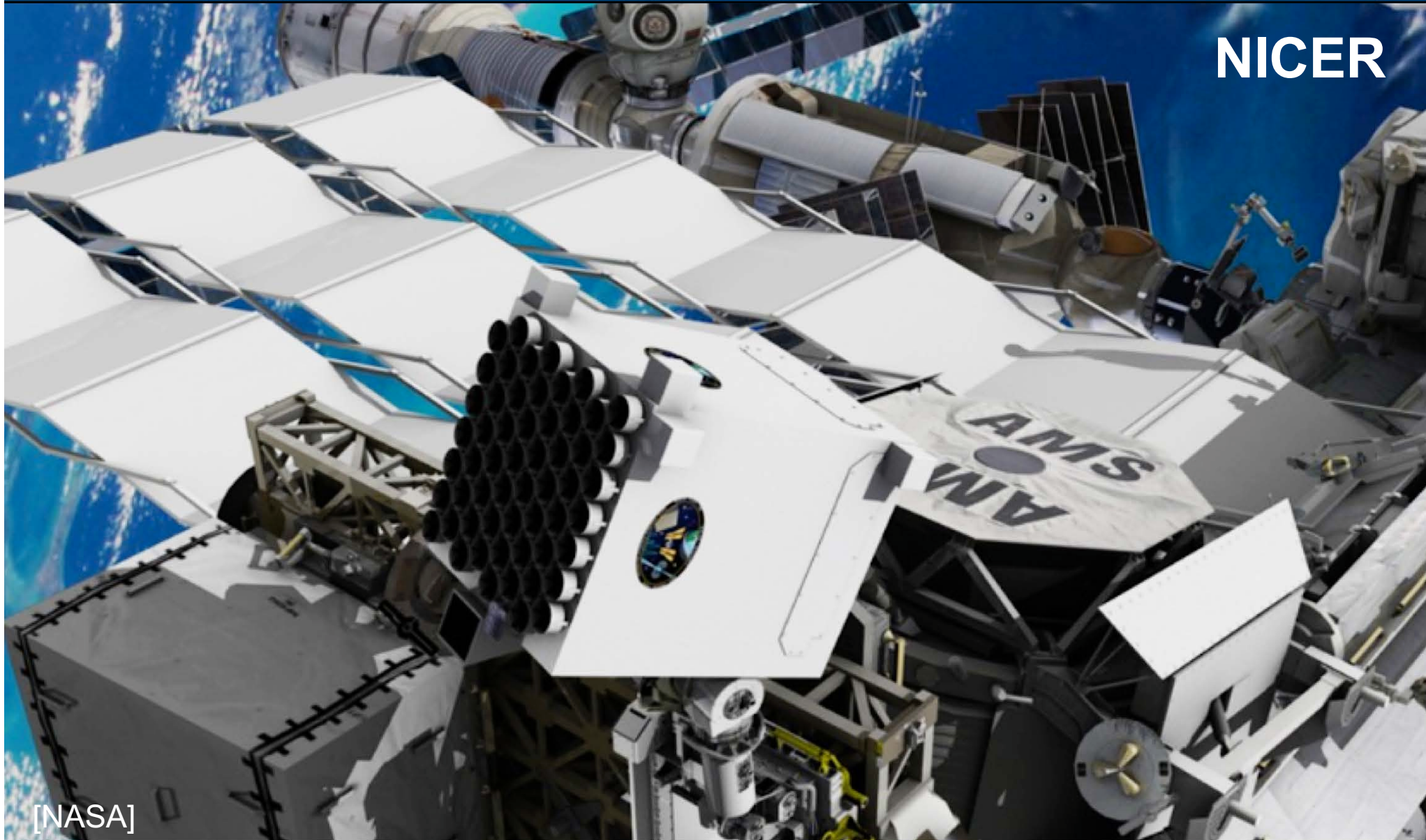
# Quantifying uncertainties in the nuclear-matter equation of state

(Simultaneous) Mass-radius constraints

Berkeley  
UNIVERSITY OF CALIFORNIA

see also Greif *et al.*, MNRAS 485, 4

NICER

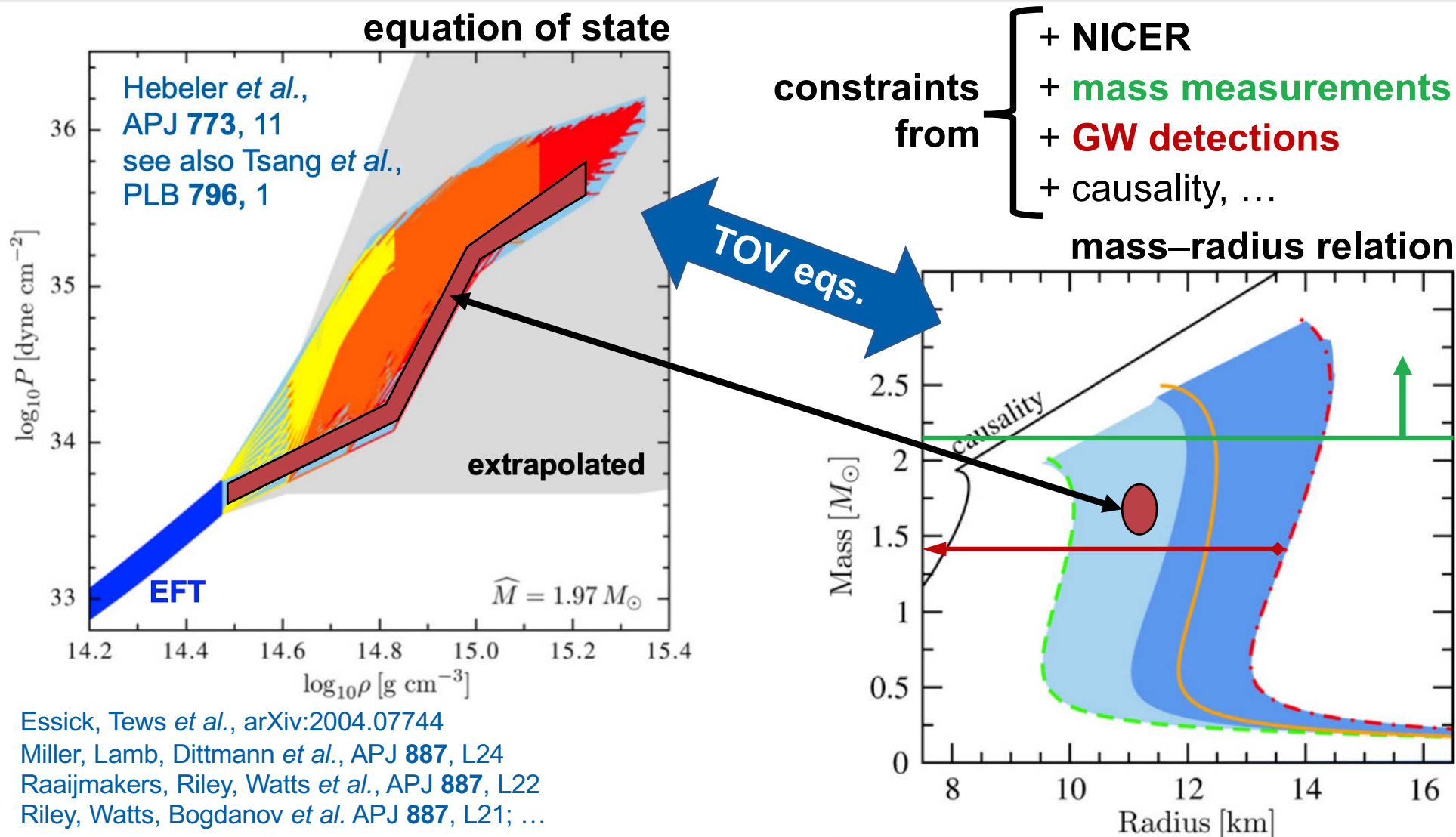


[NASA]

# Quantifying uncertainties in the nuclear-matter equation of state

(Simultaneous) Mass-radius constraints

see also Greif *et al.*, MNRAS 485, 4



Essick, Tews *et al.*, arXiv:2004.07744

Miller, Lamb, Dittmann *et al.*, APJ 887, L24

Raaijmakers, Riley, Watts *et al.*, APJ 887, L22

Riley, Watts, Bogdanov *et al.* APJ 887, L21; ...

# Quantifying uncertainties in the nuclear-matter equation of state

ISNET: Bringing statisticians and physicists together!

Berkeley  
UNIVERSITY OF CALIFORNIA

frib.msu.edu

**ISNET** Information and Statistics in Nuclear Experiment and  
Theory (ISNET 8)  
**2020**

**Facility for Rare Isotope Beams  
Michigan State University  
December 14 to 18, 2020**

## **Local Organizing Committee:**

Pawel Danielewicz  
Dean Lee  
Scott Pratt  
Frederi Viens

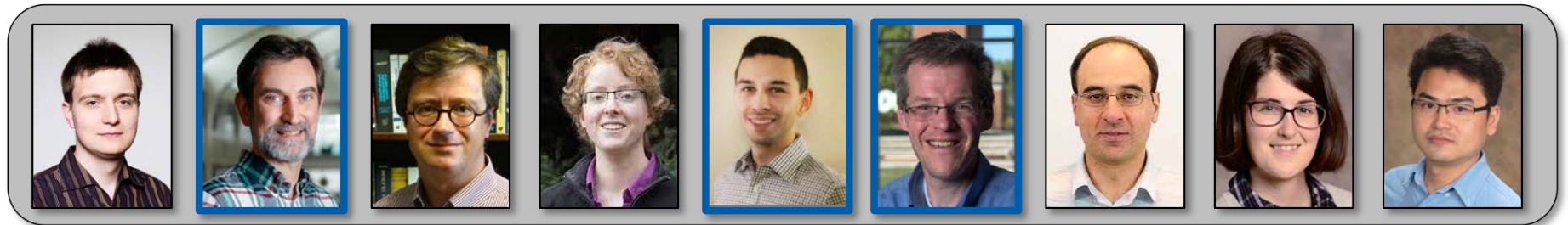
Morten Hjorth-Jensen  
Witek Nazarewicz  
Betty Tsang



*Bayesian Inference in Subatomic Physics – A Marcus Wallenberg Symposium* at Chalmers  
September 17-20, 2019, organized by: A. Ekström, C. Forssén, and S. Wesolowski  
INT Program INT-16-2a: *Bayesian Methods in Nuclear Physics*

# Quantifying uncertainties in the nuclear-matter equation of state

Thanks to my collaborators!

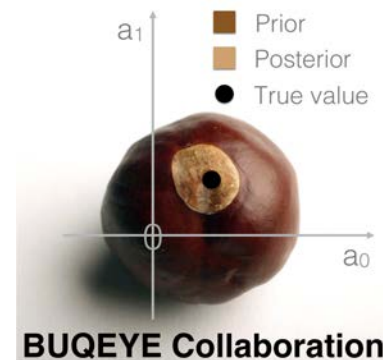


## CD, Furnstahl, Melendez, and Phillips

*How well do we know the neutron-matter equation of state at the densities inside neutron stars? A Bayesian approach with correlated uncertainties, [arXiv:2004.07232](https://arxiv.org/abs/2004.07232).*

## CD, Melendez, Furnstahl, and Phillips

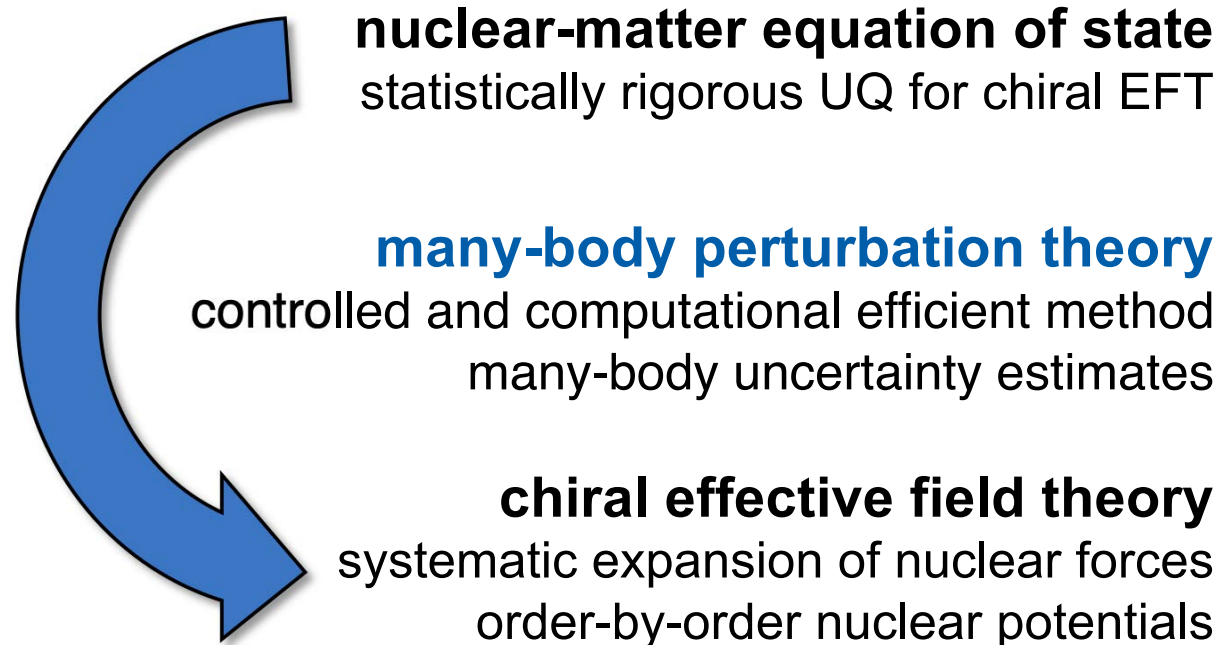
*Effective Field Theory Convergence Pattern of Infinite Nuclear Matter, [arXiv:2004.07805](https://arxiv.org/abs/2004.07805).*



**Bayesian  
Uncertainty  
Quantification:  
Errors for  
Your  
EFT**

**UQ framework available at  
<https://buqeye.github.io>**

# Quantifying uncertainties in the nuclear-matter equation of state



NPLQCD ...

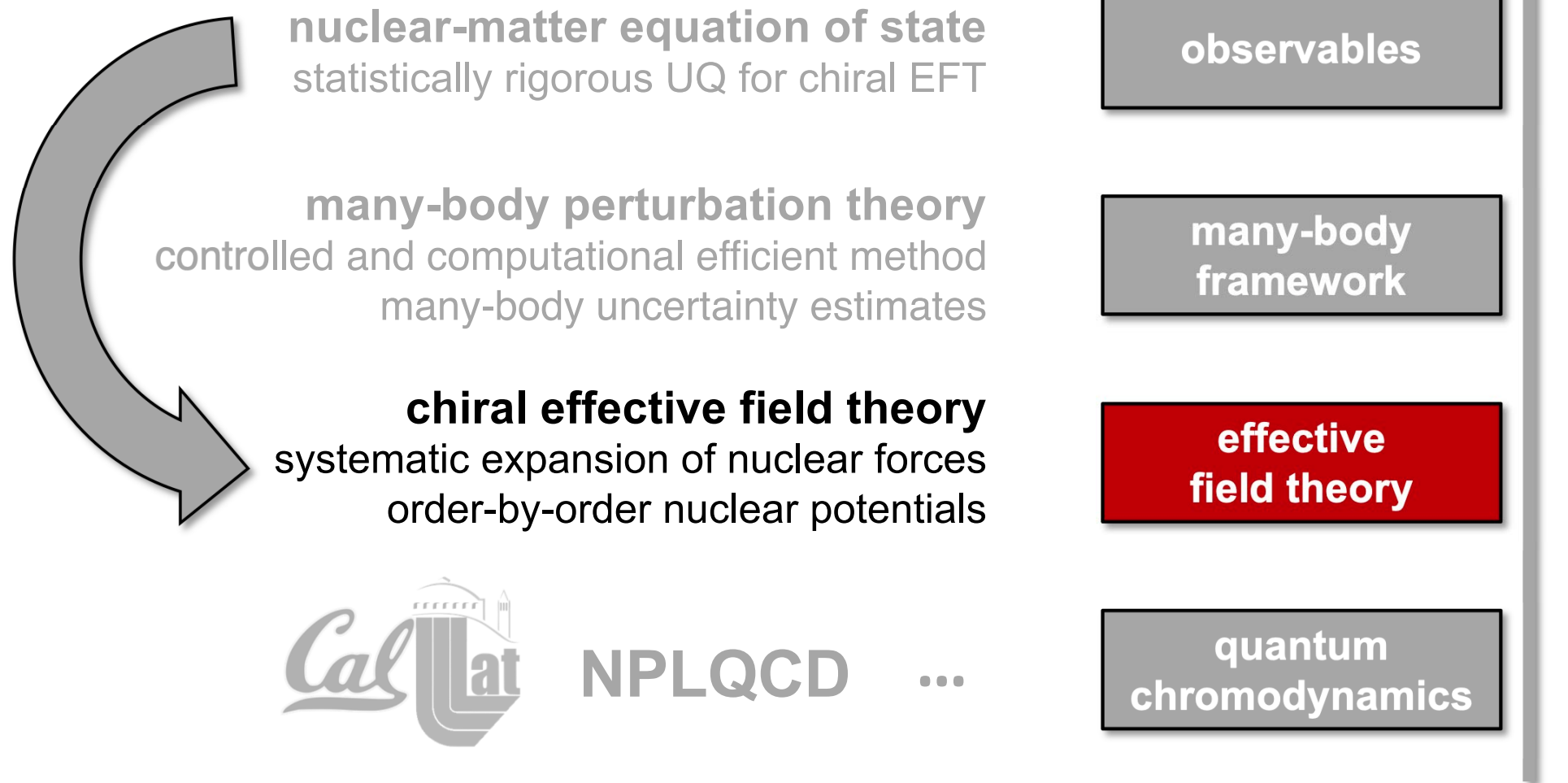
observables

many-body  
framework

effective  
field theory

quantum  
chromodynamics

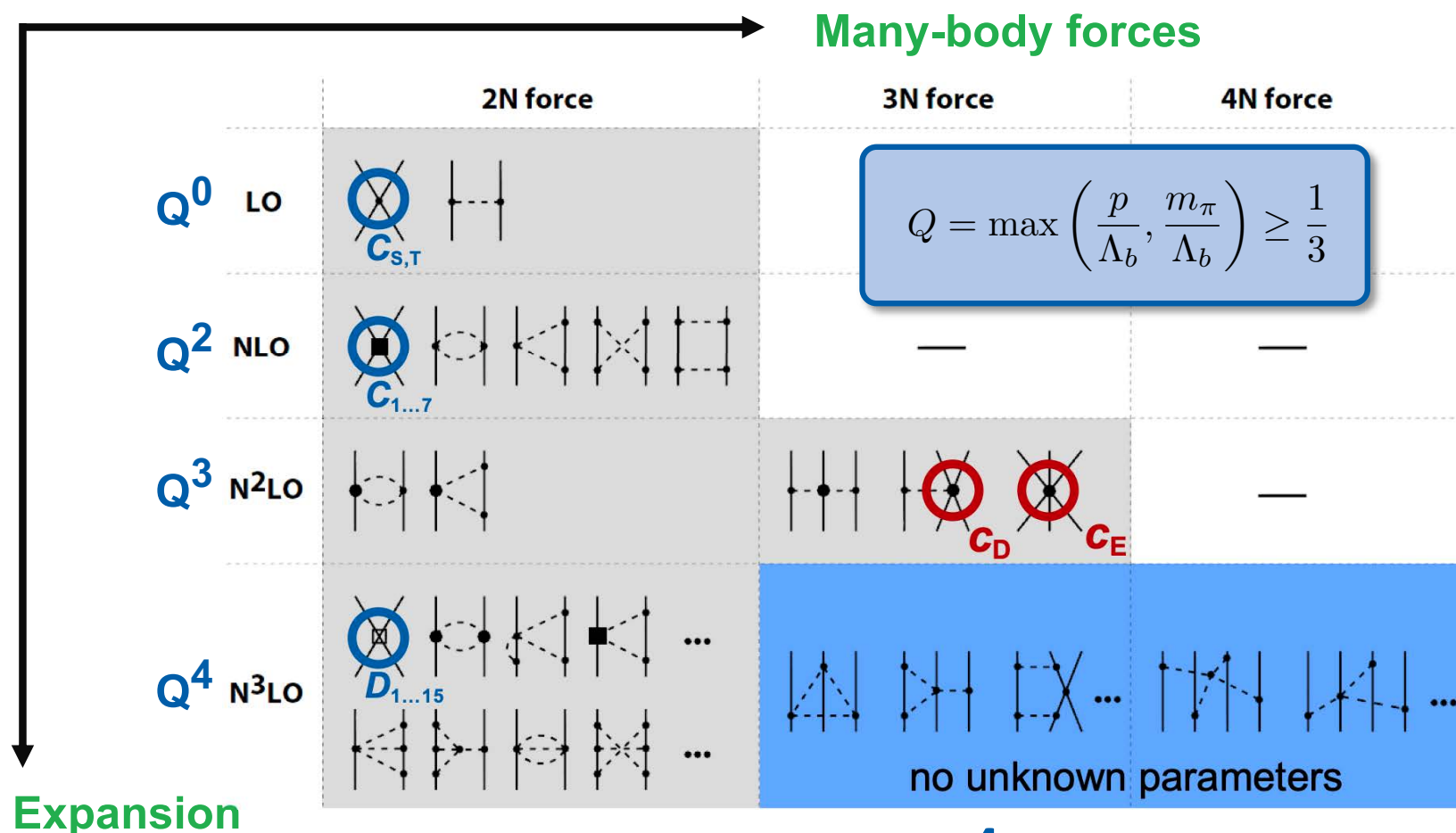
# Quantifying uncertainties in the nuclear-matter equation of state



# Quantifying uncertainties in the nuclear-matter equation of state

## Hierarchy of nuclear forces in chiral EFT

e.g., Machleidt, Entem, Phys. Rep. **503**, 1



S. Weinberg

... and ongoing work at N<sup>4</sup>LO and even N<sup>5</sup>LO...

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Krebs, Machleidt, Meißner, ...

# Quantifying uncertainties in the nuclear-matter equation of state

Including correlations in UQ is important!

$$\delta y_{i \geq 2}(x) = \max_{2 \leq j \leq i} (Q^{i+1}(x) |y_0(x)|, Q^{i+1-j}(x) |\Delta y_j(x)|)$$
$$\delta y_0(x) = Q^2(x) |y_0(x)|$$

Epelbaum, Krebs, Meißner,  
EPJ A 51, 53

$x$  : generic sampling point  
 $y_n(x)$  :  $n$ -th order prediction

The “**standard EFT**” **uncertainty** does *not* account for correlations:

**Type-x:**

Correlations between  **$y(x)$  and  $y(x')$** ; e.g.,  $y = E/A(n^{\text{th}})$ ;  
**quantified** and **propagated** using the ML  
truncation-error model proposed by BUQEYE

Melendez, Furnstahl *et al.*, PRC **100**, 044001

Melendez's GSUM, [github.com/buqeye/gsum](https://github.com/buqeye/gsum)



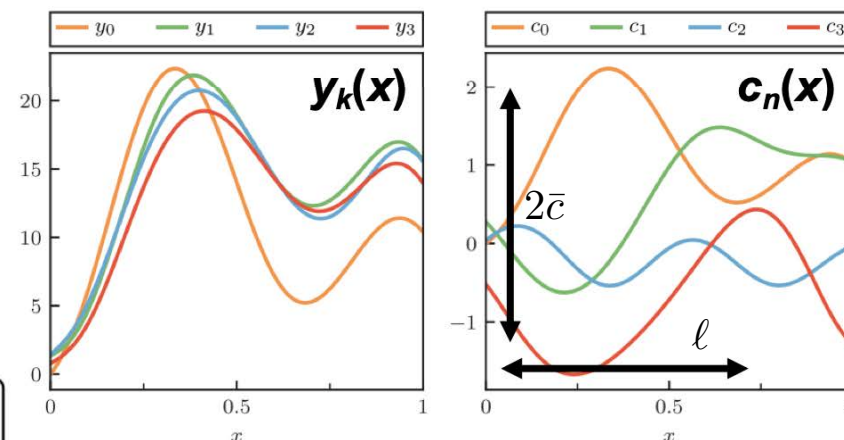
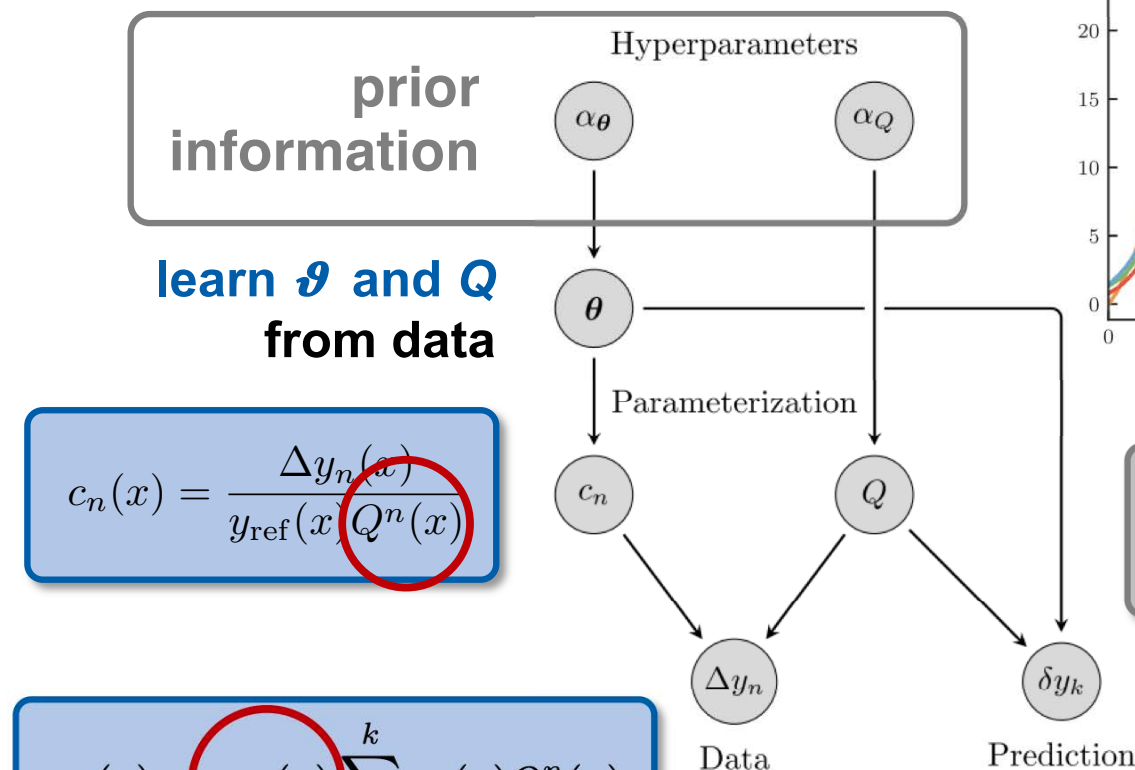
**Type-y:**

Correlations between discrete observables  **$y_i(x)$  and  $y_j(x')$** ;  
e.g., an observable  $i$  and its derivative  $j$   
extension of the truncation error model: now **quantified** and  
**propagated** using **multi-output Gaussian Processes (GPs)**

[github.com/buqeye/nuclear-matter-convergence](https://github.com/buqeye/nuclear-matter-convergence)

# Quantifying uncertainties in the nuclear-matter equation of state

BUQEYE's EFT truncation-error model



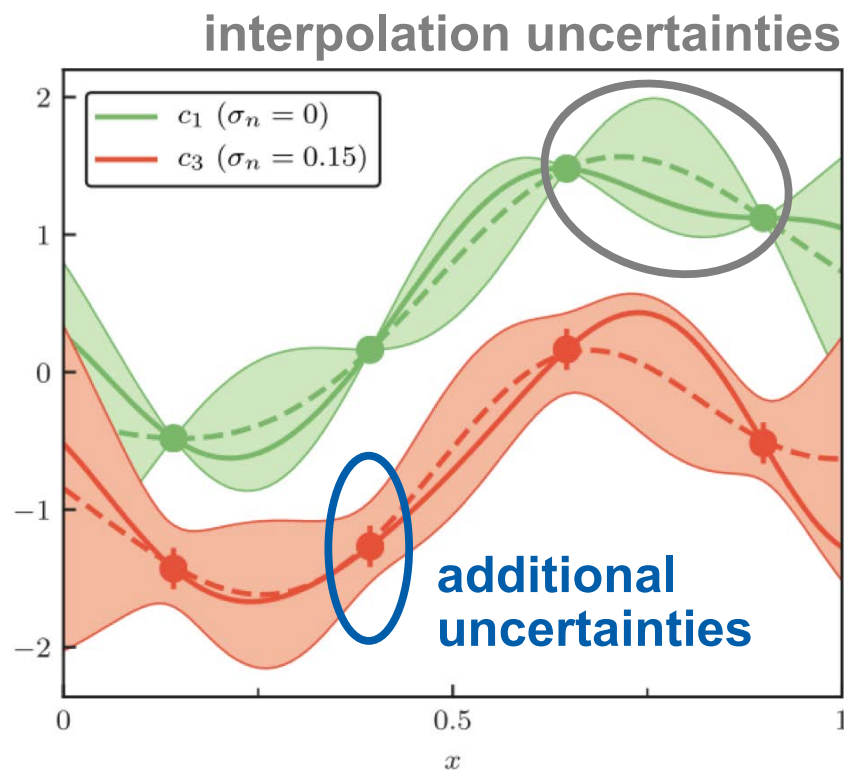
$$c_n(x) | \bar{c}^2, \ell \\ \stackrel{\text{iid}}{\sim} \mathcal{GP}[0, \bar{c}^2 r(x, x'; \ell)]$$

model all  $c_n(x)$  as independent draws from a single GP

$$\delta y_k(x) = y_{\text{ref}}(x) \sum_{n=k+1}^{\infty} c_n(x) Q^n(x)$$

# Quantifying uncertainties in the nuclear-matter equation of state

Using GPs for interpolation



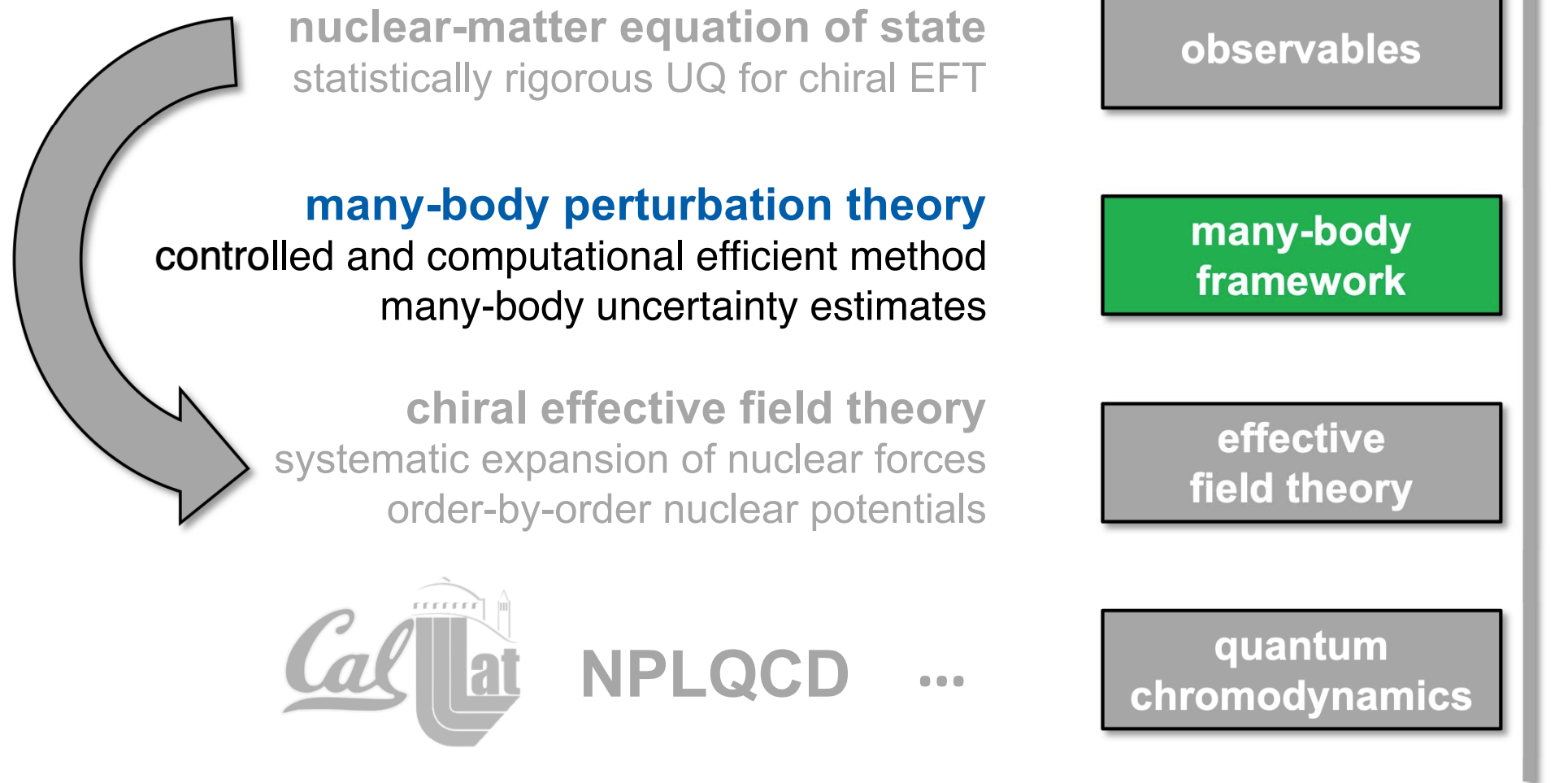
**additional** (many-body) **uncertainties** can be **easily included**

GPs are **closed** under **differentiation**  
straightforward to compute derivatives  
with correlations accounted for  
e.g., **pressure** and **speed of sound**

» **in contrast to other methods**

**Gaussian Process** interpolates the EOS  
while quantifying uncertainties, including  
**to-all-orders EFT truncation errors**

# Quantifying uncertainties in the nuclear-matter equation of state



# Quantifying uncertainties in the nuclear-matter equation of state

Efficient Monte Carlo framework



**efficient evaluation** of **MBPT diagrams**  
with **NN**, **3N**, and **4N forces** (single-particle basis)

- **implementing diagrams** has become **straightforward** (incl. particle-hole terms)
- multi-dimensional momentum integrals: (improved) Lepage's VEGAS algorithm
- **controlled computation** of arbitrary interaction and many-body diagrams

**EOS** up to  
high orders



**automatic code  
generation**



**analytic form  
of the diagrams**

# Quantifying uncertainties in the nuclear-matter equation of state

Significant challenges are past!



## Higher orders: particle-hole contributions

Coraggio *et al.*, PRC **89**, 044321; Holt, Kaiser, PRC **95**, 034326



## No approximations for 3N normal ordering

CD *et al.*, PRC **93**, 054314; Holt *et al.*, PRC **81**, 024002



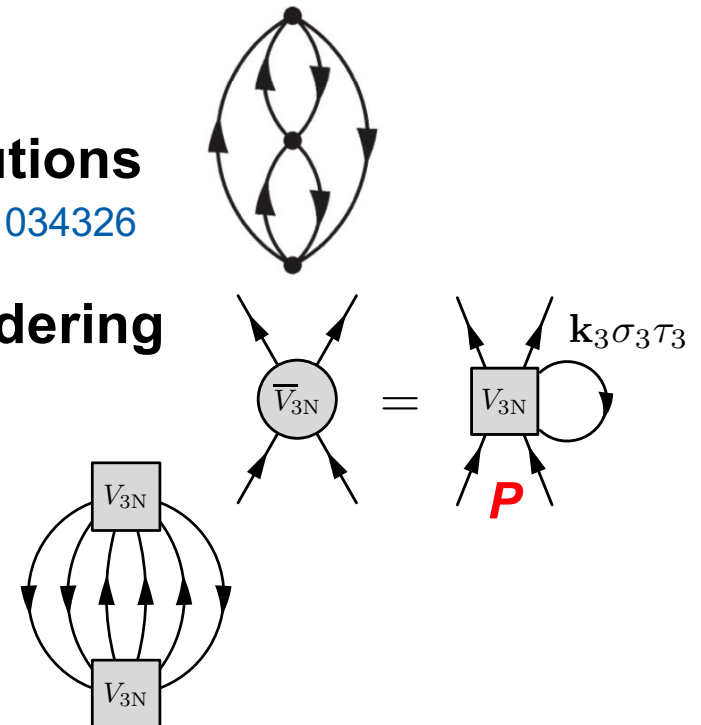
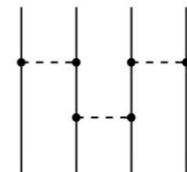
## Include residual 3N diagram(s)

Hagen *et al.*, PRC **89**, 014319; Kaiser, EPJ A **48**, 58



## Higher many-body forces

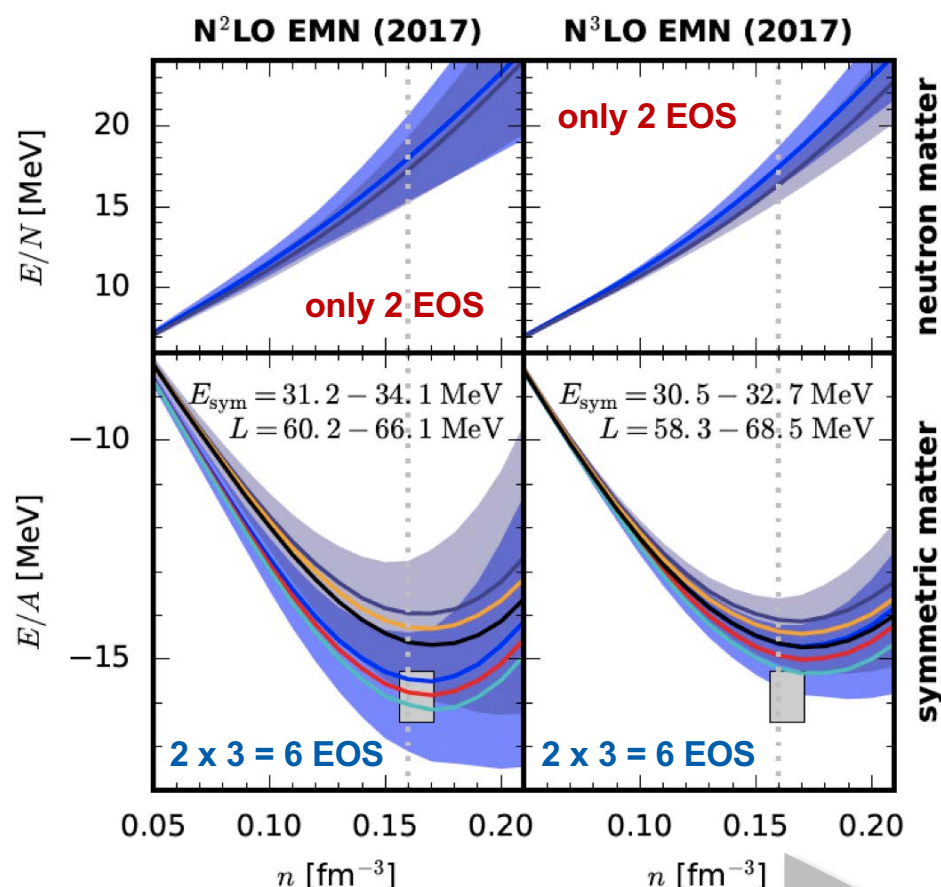
Hebeler *et al.*, PRC **91**, 044001



development of a novel  
**Monte Carlo** framework

# Quantifying uncertainties in the nuclear-matter equation of state

Many-body input of our UQ



extended up to  $2n_0$

use the Monte Carlo framework to evaluate the EOS up to N<sup>3</sup>LO

bare N<sup>2</sup>LO / N<sup>3</sup>LO EMN potentials

Entem, Machleidt, Nosyk, PRC **96**, 024004

Hoppe, CD, Furnstahl *et al.*, PRC **96**, 054002

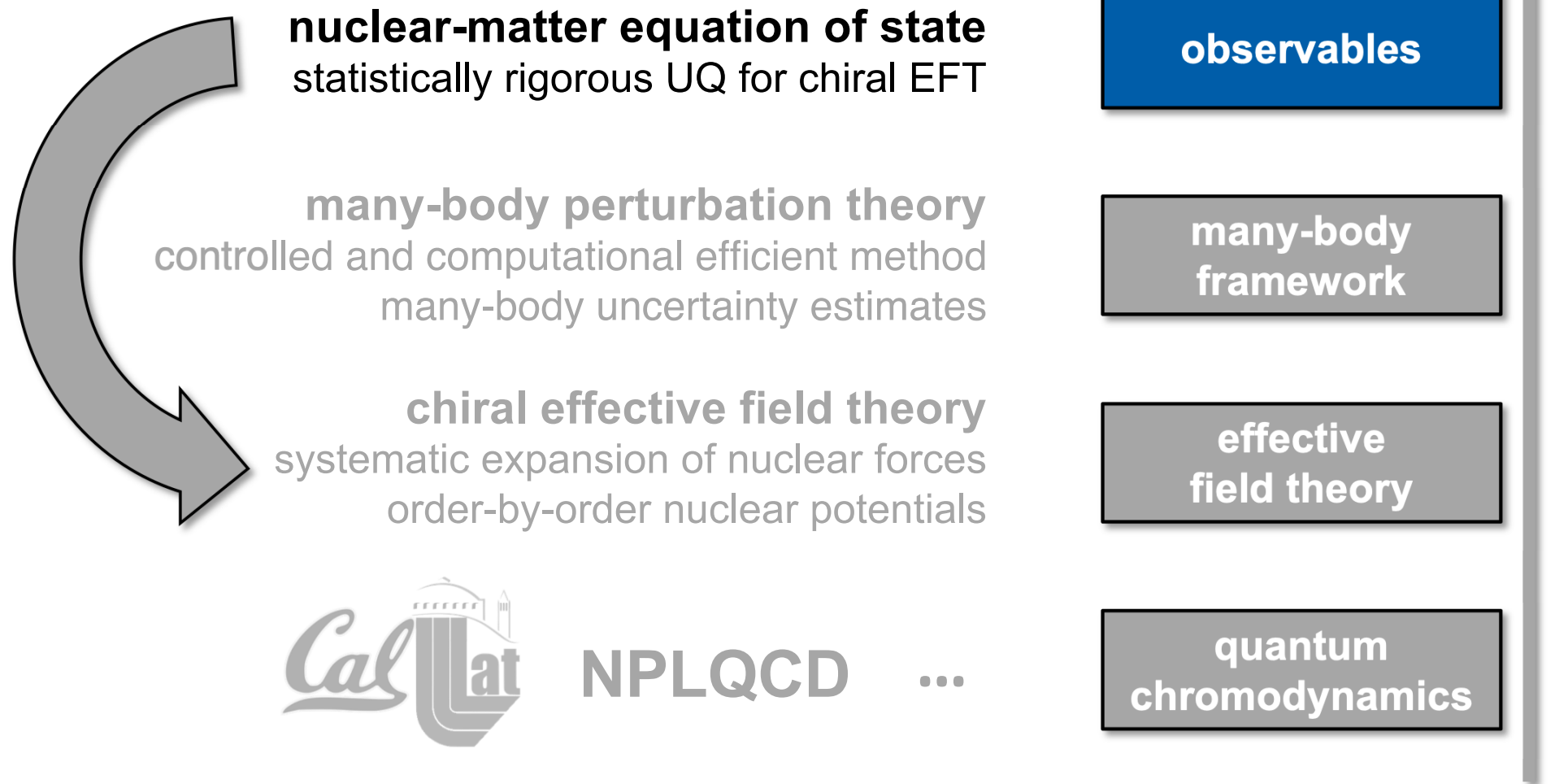
fit 3N interactions to the triton and nuclear **saturation point**

identified  $(c_D, c_E)$  which **saturate** close to the empirical **saturation point**

3 Hamiltonians available for each cutoff and chiral order; **in total: 12**

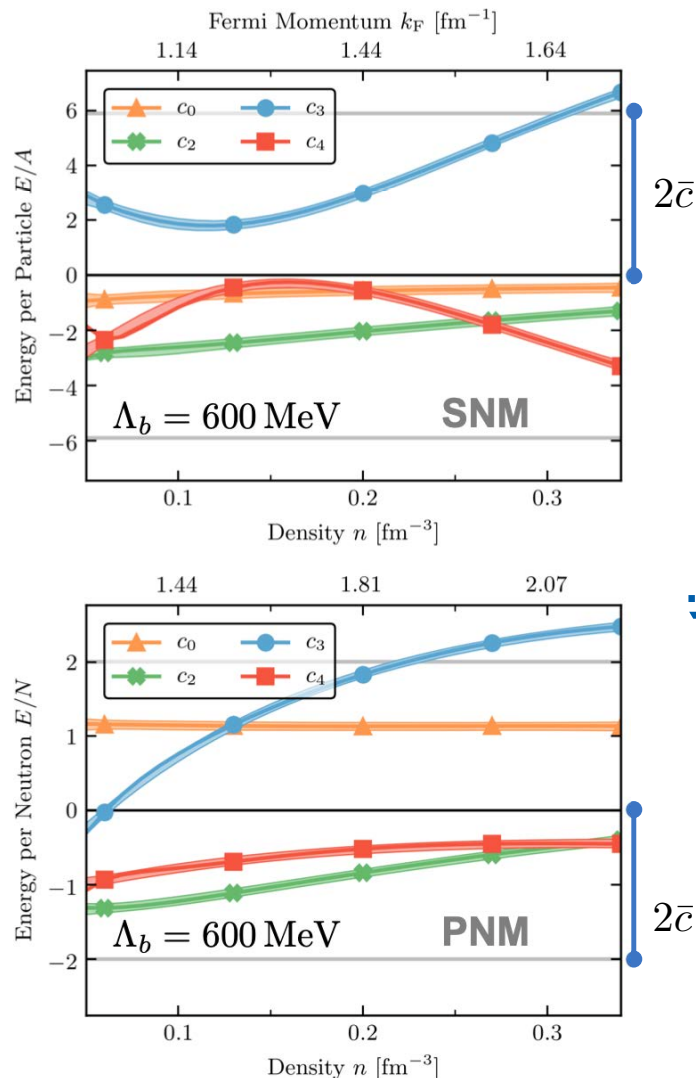
| $\Lambda/c_D$ [MeV]/[1] |           | $\Lambda/c_D$ [MeV]/[1] |           |
|-------------------------|-----------|-------------------------|-----------|
| 450/2.25                | 500/-1.75 | 450/0.00                | 500/-3.00 |
| 450/2.50                | 500/-1.50 | 450/0.25                | 500/-2.75 |
| 450/2.75                | 500/-1.25 | 450/0.50                | 500/-2.50 |

# Quantifying uncertainties in the nuclear-matter equation of state



# Quantifying uncertainties in the nuclear-matter equation of state

Extracting the observable coefficients



We need to choose:

$$c_n(x) = \frac{\Delta y_n(x)}{y_{\text{ref}}(x) Q^n(x)}$$

**$\mathbf{x}$**  :  $\gamma k_F$  or  $n = g \frac{k_F^3}{6\pi^2}$   $\gamma > 0$   
stationary kernel: same curviness across entire range

**$y_{\text{ref}}(\mathbf{x})$**  :  $y_{\text{ref}}(k_F) = 16 \text{ MeV} \times \left( \frac{\gamma k_F}{k_{F,0}} \right)^2$   
naturalness: overall scale for  $c_n(\mathbf{x})$

**$Q(\mathbf{x})$**  :  $Q(k_F) = \frac{p}{\Lambda_b}$   $p = \gamma k_F$   
What is the soft scale of nuclear matter?

**Trust, but verify**  
model-checking diagnostics  $\gamma = 1$

feedback

# Quantifying uncertainties in the nuclear-matter equation of state

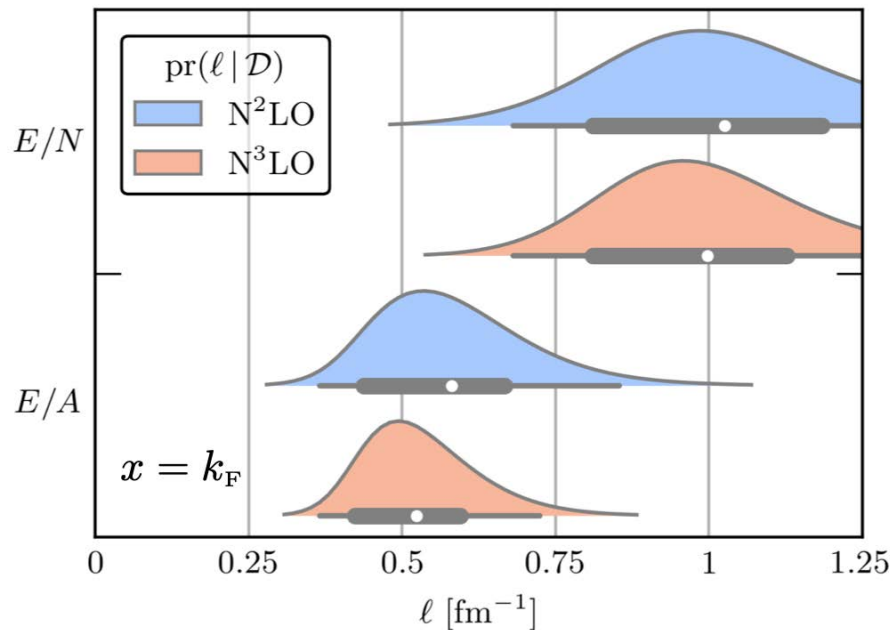
Bayesian inference from the data

How correlated  
is nuclear matter ?

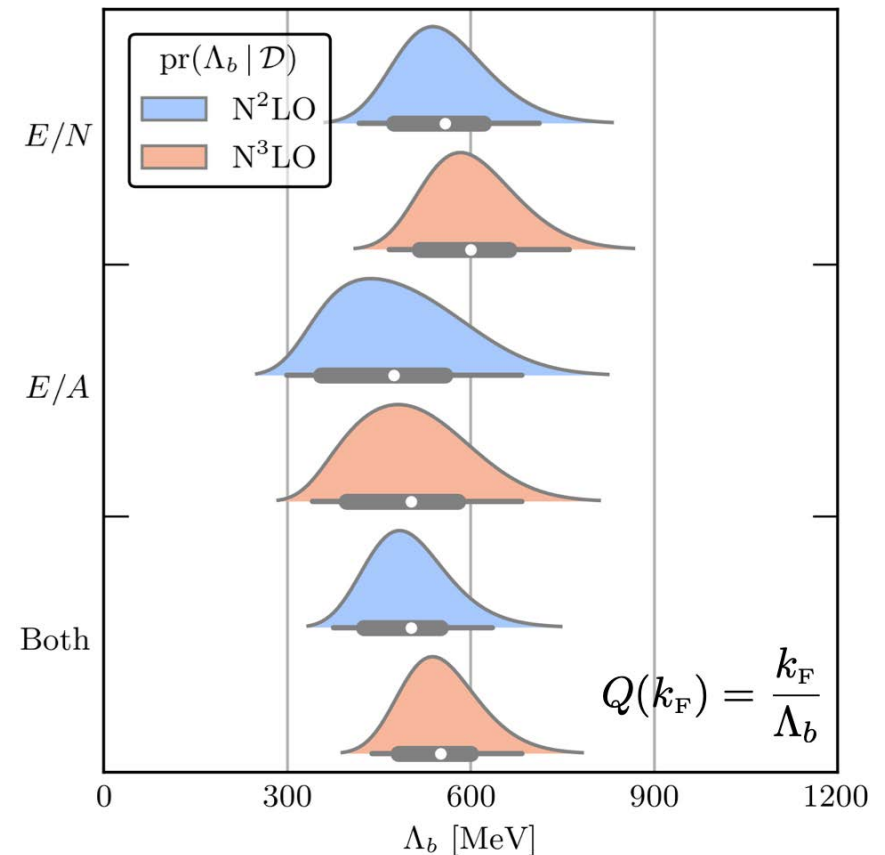
$\text{pr}(\ell | \mathcal{D})$   
correlation length

Where does the  
EFT break down ?

$\text{pr}(\Lambda_b | \mathcal{D})$   
breakdown scale



to be  
compared with

$$k_F^{\max} = \begin{cases} 2.2 \text{ fm}^{-1} & \text{PNM} \\ 1.7 \text{ fm}^{-1} & \text{SNM} \end{cases}$$


# Quantifying uncertainties in the nuclear-matter equation of state

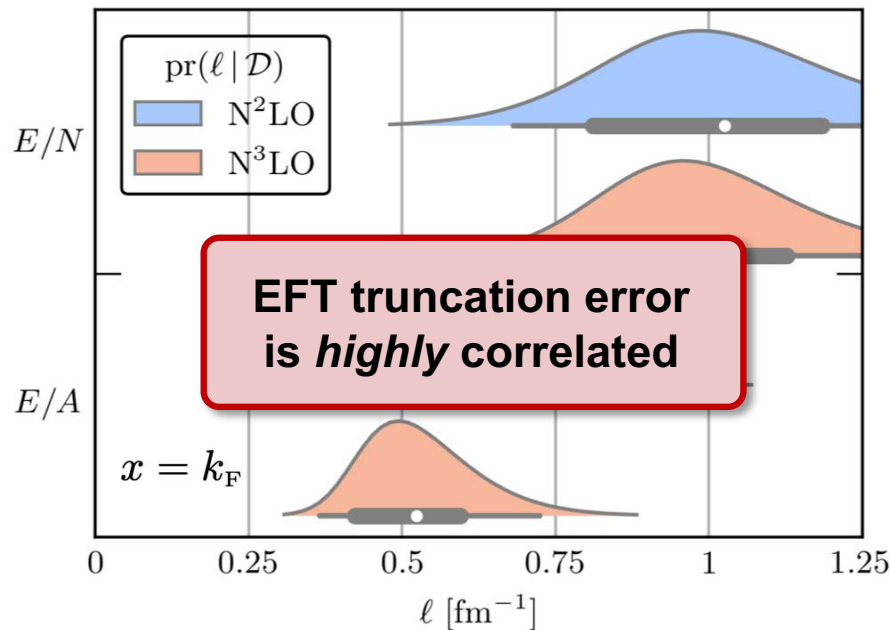
Bayesian inference from the data

How correlated  
is nuclear matter ?

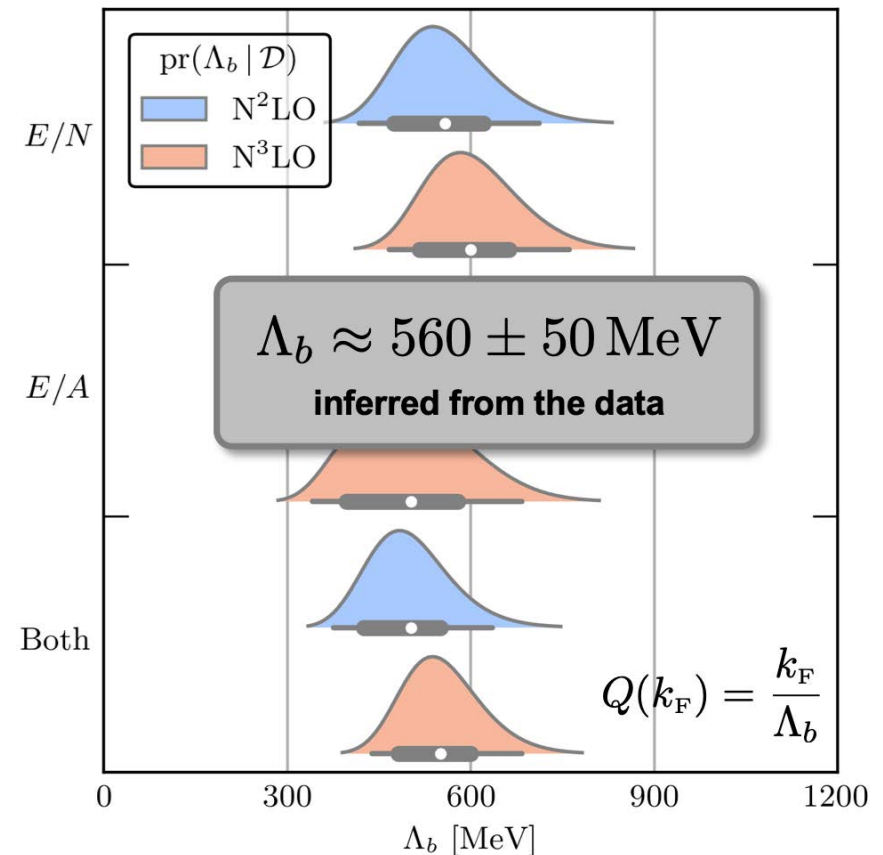
$\text{pr}(\ell | \mathcal{D})$   
correlation length

Where does the  
EFT break down ?

$\text{pr}(\Lambda_b | \mathcal{D})$   
breakdown scale



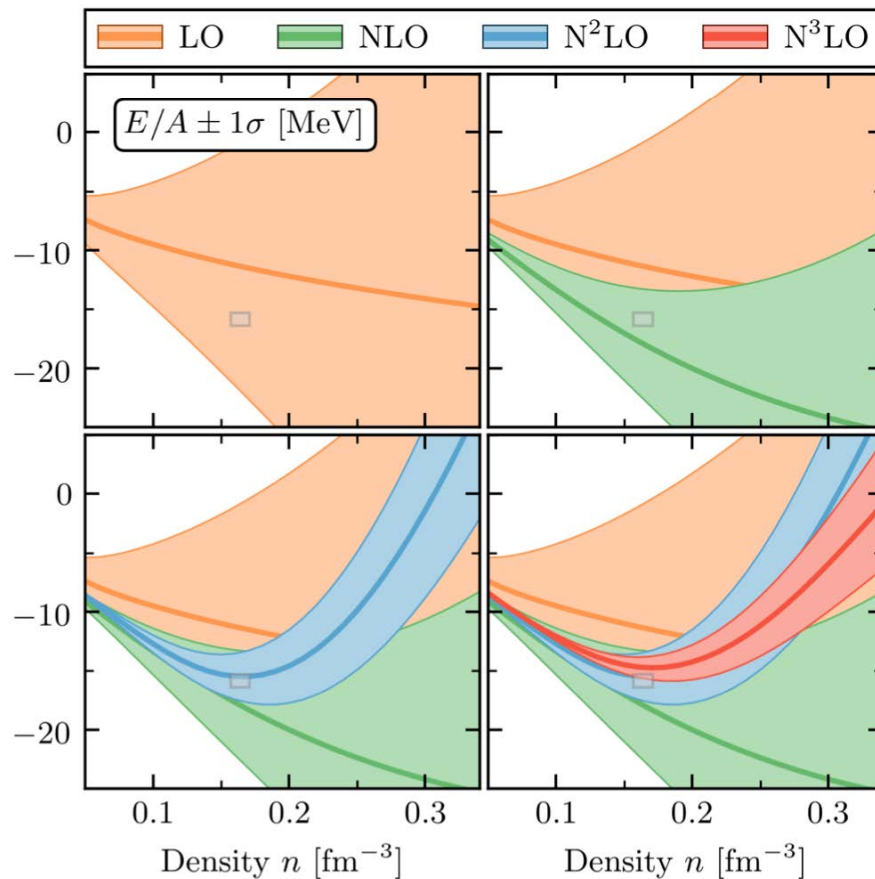
to be  
compared with

$$k_F^{\max} = \begin{cases} 2.2 \text{ fm}^{-1} & \text{PNM} \\ 1.7 \text{ fm}^{-1} & \text{SNM} \end{cases}$$


# Quantifying uncertainties in the nuclear-matter equation of state

SNM and the saturation point

$\Lambda = 450$  MeV



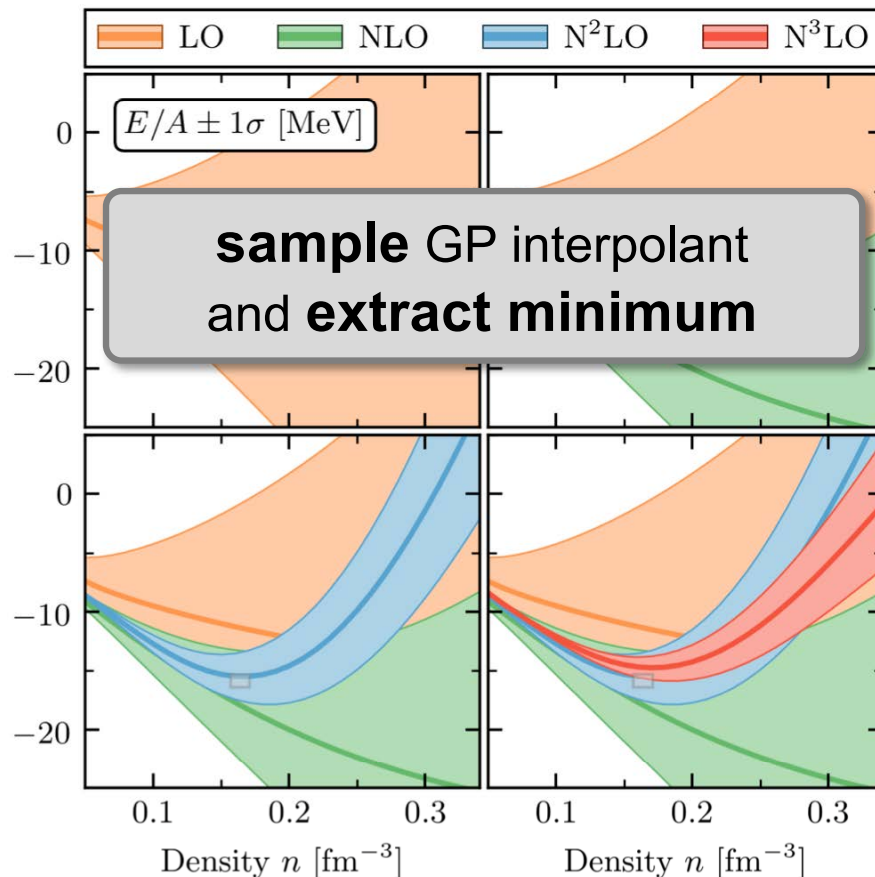
$$\text{pr} \left( \frac{E}{A}(n_0), n_0 \mid \mathcal{D} \right)$$

see also CD, Hebeler, Schwenk, PRL **122**, 042501

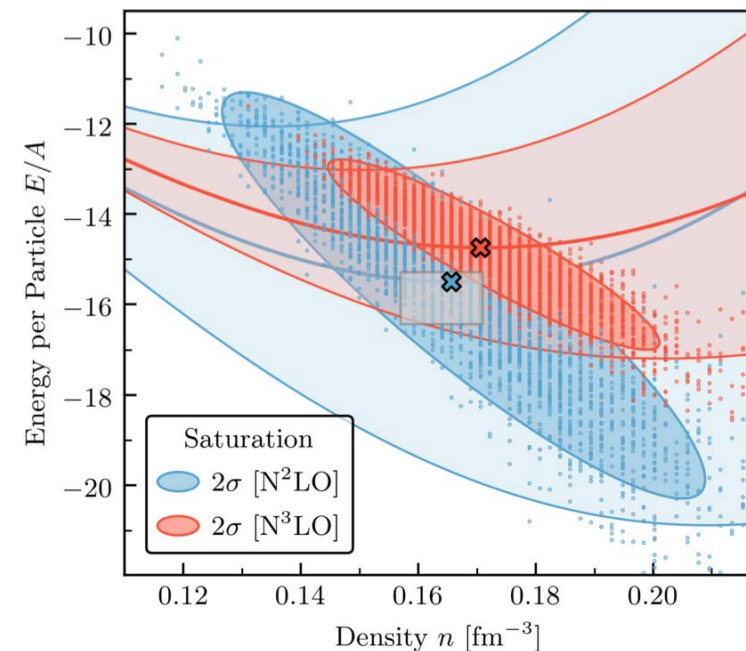
# Quantifying uncertainties in the nuclear-matter equation of state

SNM and the saturation point

$\Lambda = 450$  MeV



$$\text{pr} \left( \frac{E}{A}(n_0), n_0 \mid \mathcal{D} \right)$$



two-dimensional Gaussian

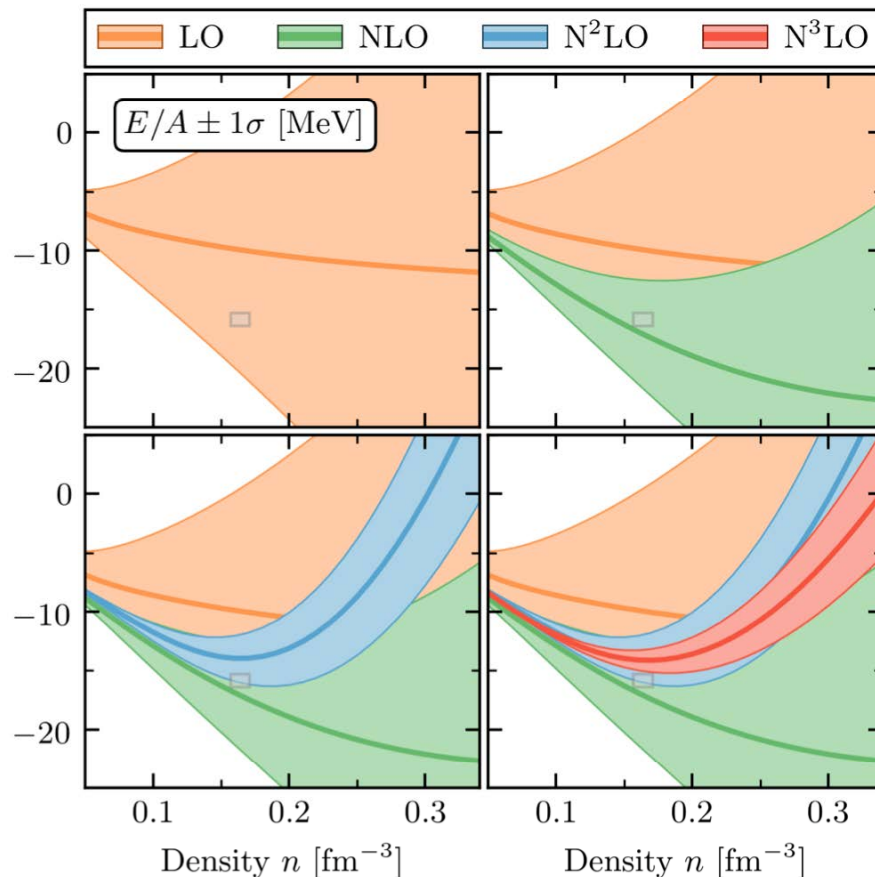
$$\begin{bmatrix} n_0 \\ \frac{E}{A}(n_0) \end{bmatrix} \approx \begin{bmatrix} 0.173 \\ -14.9 \end{bmatrix} \quad \Sigma \approx \begin{bmatrix} 0.014^2 & -0.014 \\ -0.014 & 1.1^2 \end{bmatrix}$$

see also CD, Hebeler, Schwenk, PRL **122**, 042501

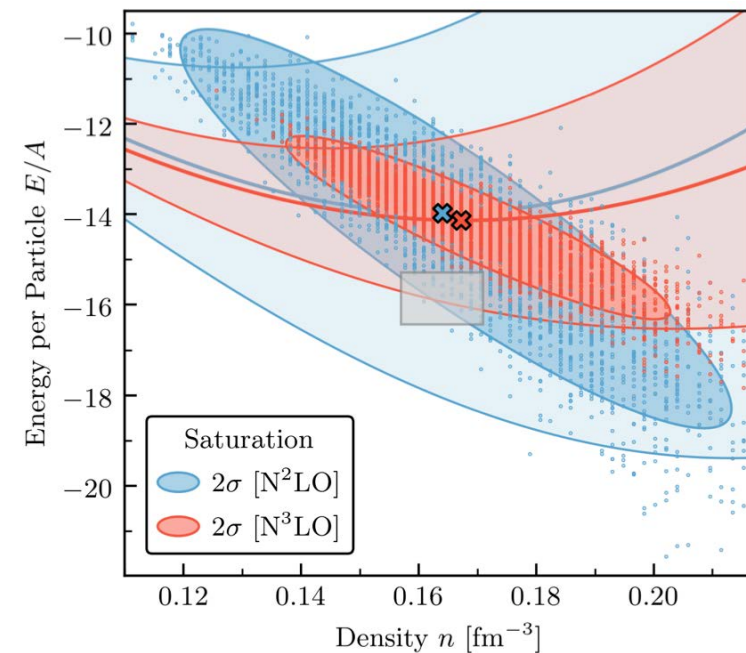
# Quantifying uncertainties in the nuclear-matter equation of state

SNM and the saturation point

$\Lambda = 500 \text{ MeV}$



$$\text{pr} \left( \frac{E}{A}(n_0), n_0 \mid \mathcal{D} \right)$$



two-dimensional Gaussian

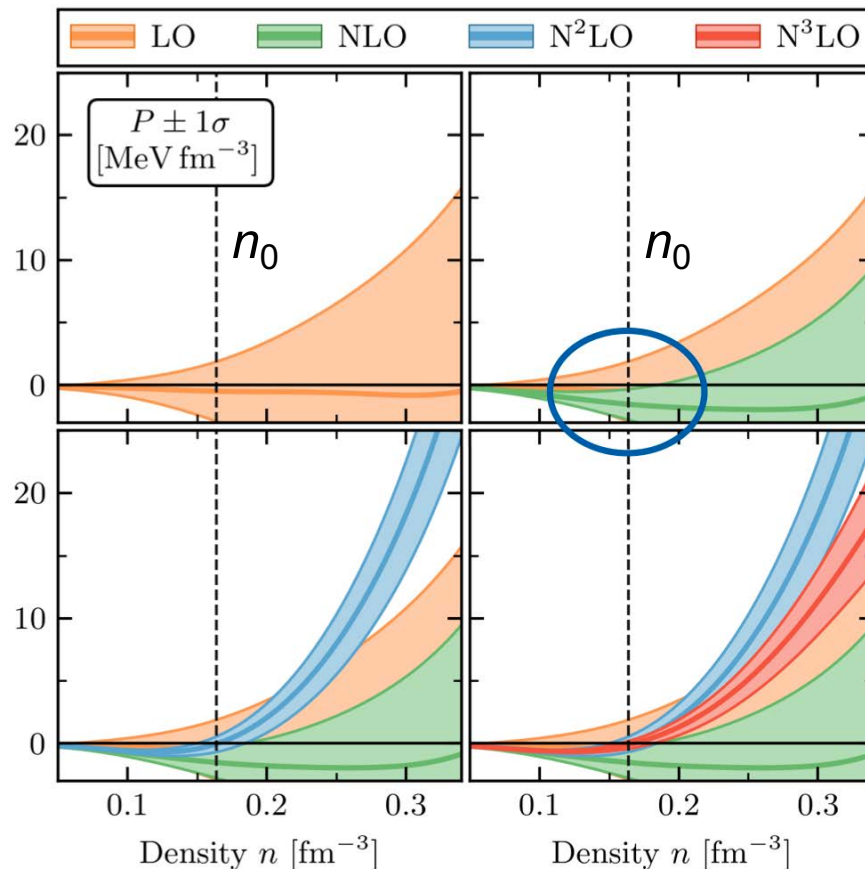
$$\begin{bmatrix} n_0 \\ \frac{E}{A}(n_0) \end{bmatrix} \approx \begin{bmatrix} 0.170 \\ -14.3 \end{bmatrix} \quad \Sigma \approx \begin{bmatrix} 0.016^2 & -0.015 \\ -0.015 & 1.0^2 \end{bmatrix}$$

see also CD, Hebeler, Schwenk, PRL **122**, 042501

# Quantifying uncertainties in the nuclear-matter equation of state

## Pressure of SNM

$\Lambda = 500 \text{ MeV}$



$$P(n) = n^2 \frac{d}{dn} \frac{E}{A}(n)$$

**! LO / NLO:** nuclear saturation at  $\sim n_0$  could be achieved within the large uncertainties

possible zero-crossing over a wide range of densities suggests that the nuclear **saturation point is *fine tuned***

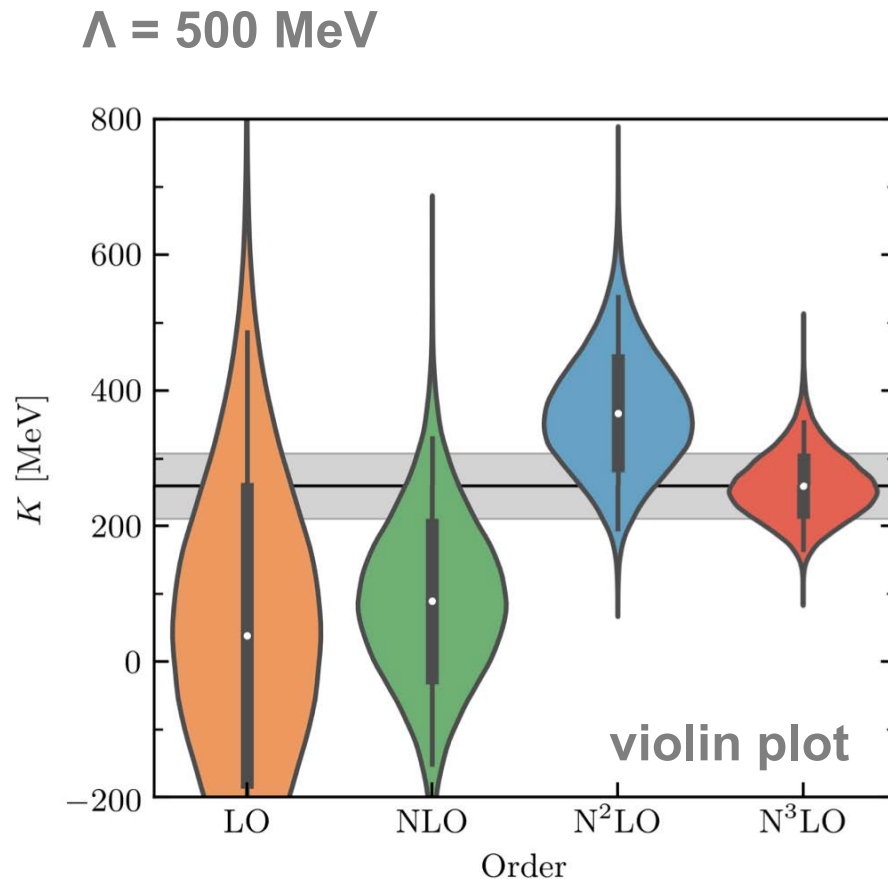
**N<sup>2</sup>LO / N<sup>3</sup>LO:** consistently within the bands of the previous orders,  $n \lesssim n_0$ , but diverge beyond

**be careful:** bands are difficult to gauge due to the *long* correlation length (small effective sample size)

» **model-checking diagnostics**

# Quantifying uncertainties in the nuclear-matter equation of state

## Incompressibility of SNM



$$K = 9n_0^2 \left. \frac{d^2 E}{dn^2} \frac{1}{A}(n) \right|_{n=n_0}$$

! bands are consistent across orders

$$\text{pr}(K | \mathcal{D}) = \int \text{pr}(K | \mathcal{D}, n_0) \text{pr}(n_0 | \mathcal{D}) dn_0$$

$$\text{pr}(n_0 | \mathcal{D}) \approx 0.17 \pm 0.01 \text{ fm}^{-3}$$

**LO / NLO:** the empirical saturation point is typically *not* well reproduced...

... leading to **wide-spread distributions** whose  $1\sigma$  regions reach  $K < 0$ , even though saturation requires  $K > 0$

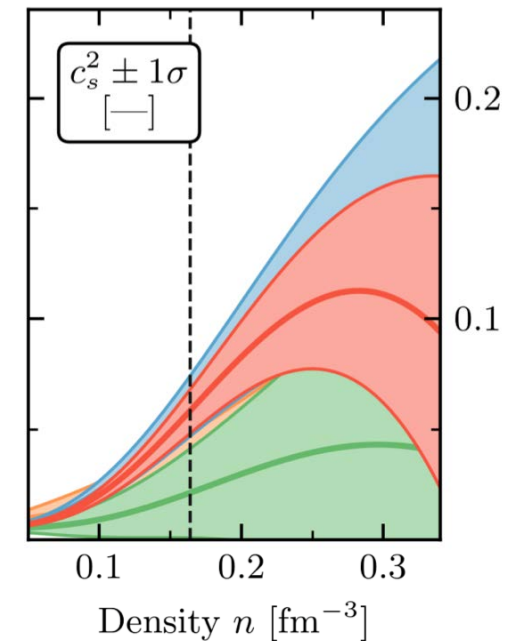
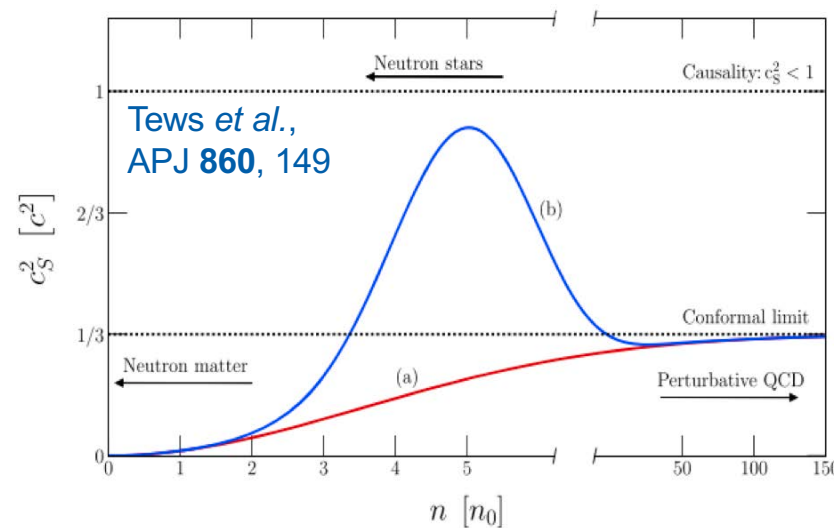
$$K = 260 \pm 54 \text{ MeV} \quad (\Lambda = 500 \text{ MeV})$$

$$K = 292 \pm 54 \text{ MeV} \quad (\Lambda = 450 \text{ MeV})$$

# Quantifying uncertainties in the nuclear-matter equation of state

Speed of sound and pressure in PNM

$$c_s^2(n) = \frac{\partial P(n)}{\partial \varepsilon(n)}$$

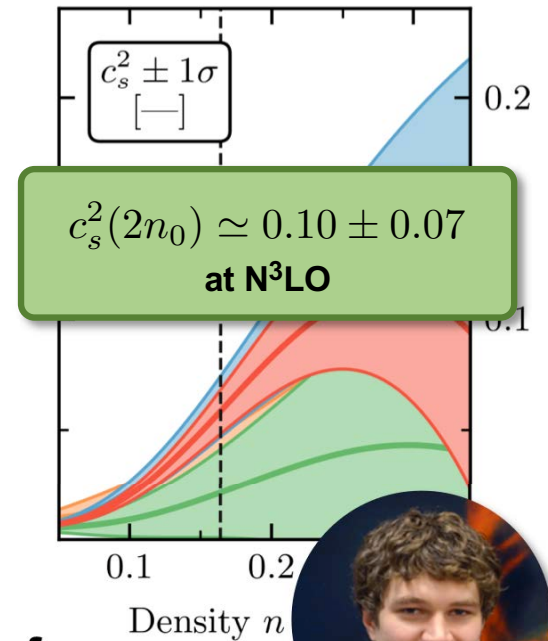
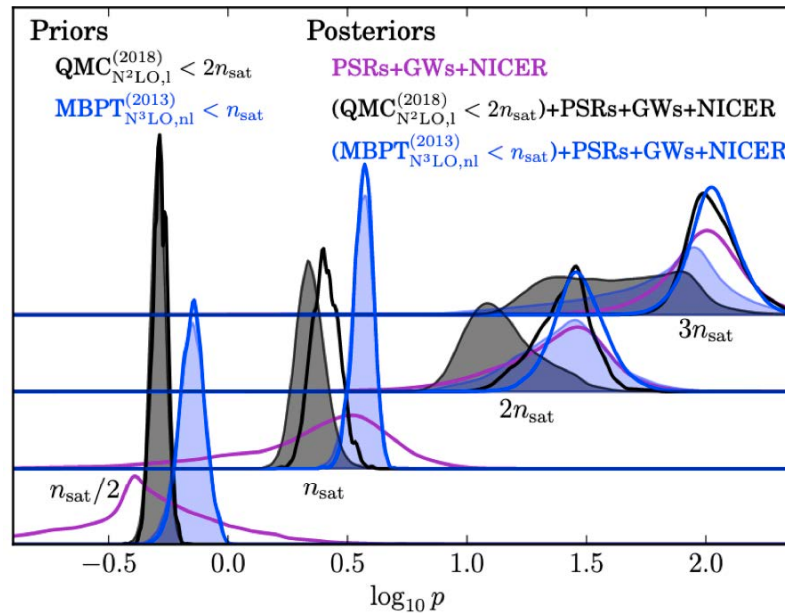
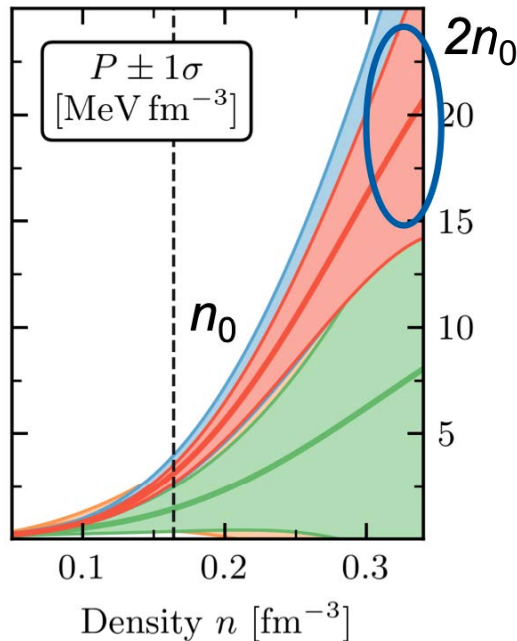


# Quantifying uncertainties in the nuclear-matter equation of state

Speed of sound and pressure in PNM

$$P(n = 0.32 \text{ fm}^{-3}) \approx \begin{cases} 20 \pm 6 \text{ MeV fm}^{-3} & \text{MBPT: nonlocal} \\ 15 \pm 5 \text{ MeV fm}^{-3} & \text{QMC: local } V_{E,1} \end{cases}$$

$$c_s^2(n) = \frac{\partial P(n)}{\partial \varepsilon(n)}$$



$$P(n) = n^2 \frac{d}{dn} \frac{E}{N}(n)$$

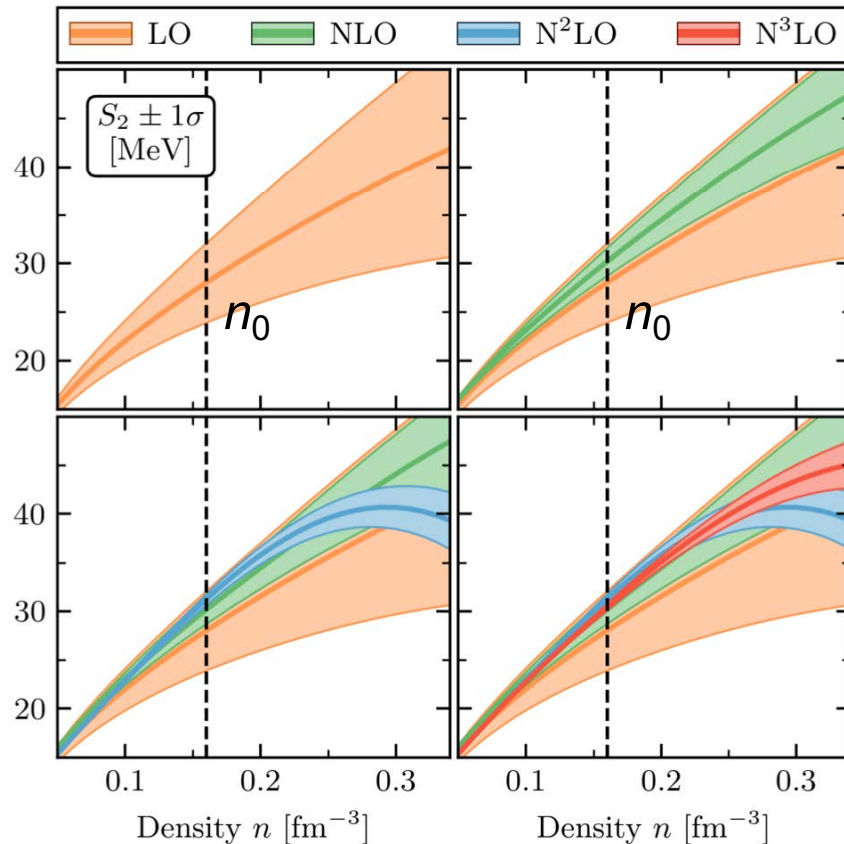
Direct astrophysical tests of  
chiral EFT at supranuclear densities

Essick, Tews, Landry, Reddy, Holz, arXiv:2004.07744



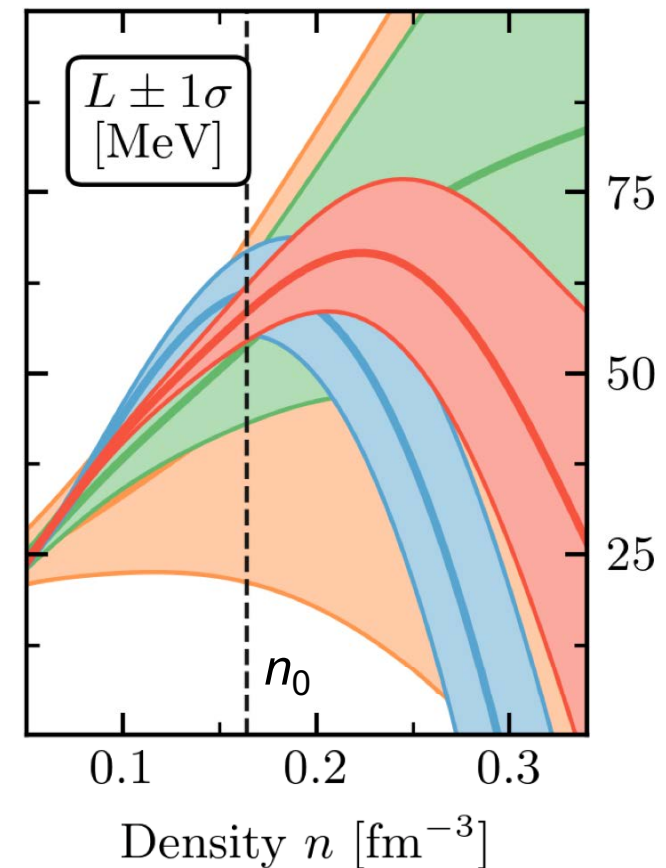
# Quantifying uncertainties in the nuclear-matter equation of state

## Nuclear symmetry energy



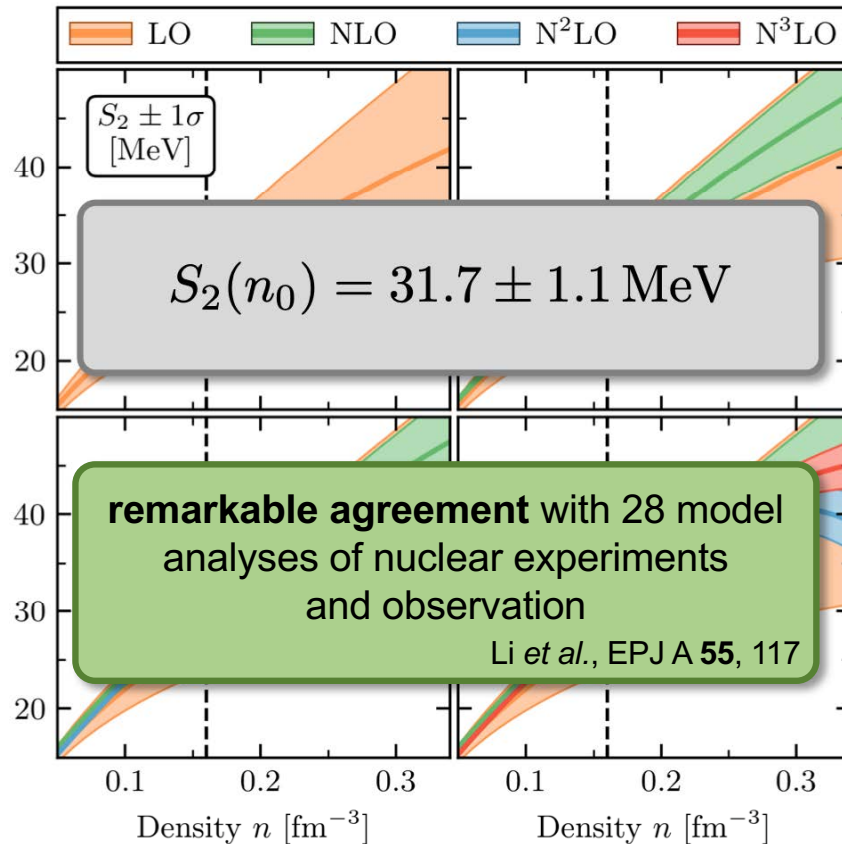
$$S_2(n) \approx \frac{E}{N}(n) - \frac{E}{A}(n)$$

$$L(n) \equiv 3n \frac{d}{dn} S_2(n)$$



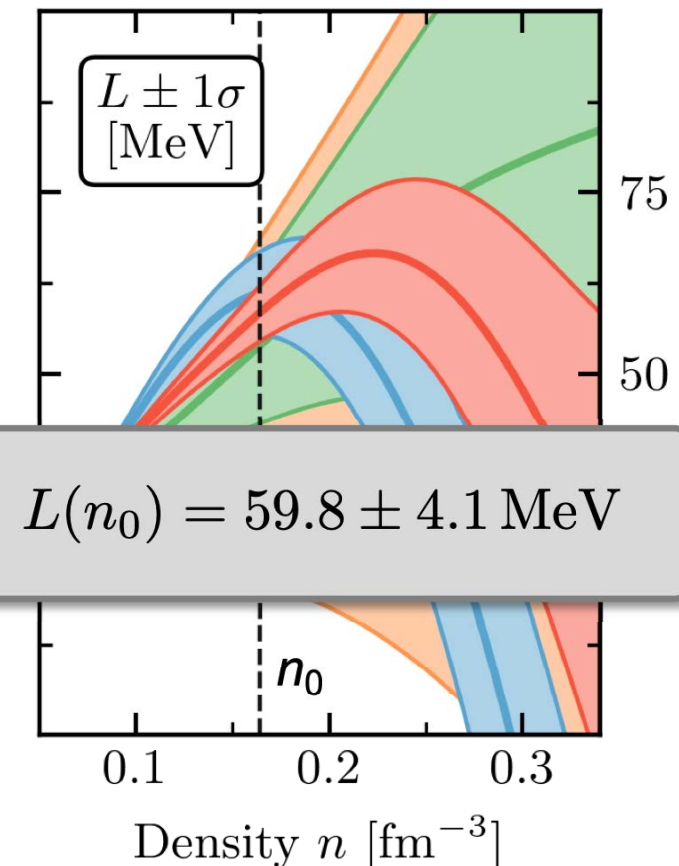
# Quantifying uncertainties in the nuclear-matter equation of state

## Nuclear symmetry energy



$$S_2(n) \approx \frac{E}{N}(n) - \frac{E}{A}(n)$$

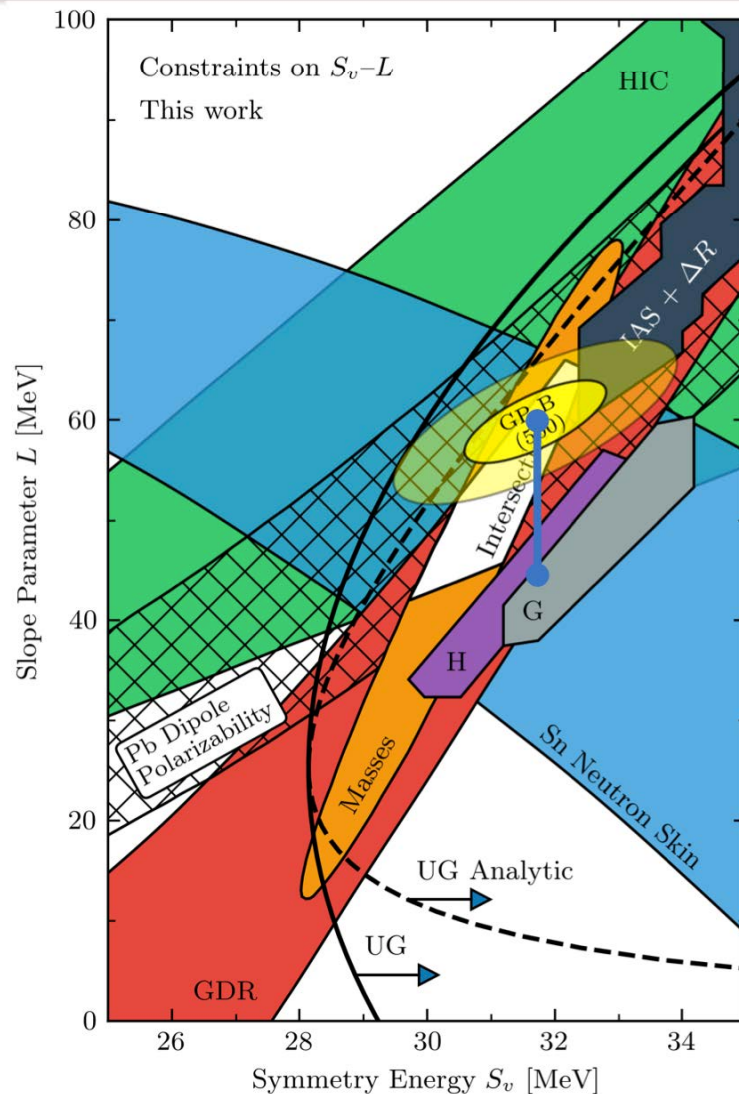
$$L(n) \equiv 3n \frac{d}{dn} S_2(n)$$



# Quantifying uncertainties in the nuclear-matter equation of state

$S_v$ - $L$  correlation (as compiled by Lattimer *et al.*)

CD, Furnstahl *et al.*, arXiv:2004.07805



$$S_2(n) \equiv S_v + \frac{L}{3} \left( \frac{n - n_0}{n_0} \right) + \dots$$

**Excellent agreement with experiment**

Lattimer and Lim, APJ 771, 51

$$\text{pr}(S_v, L | \mathcal{D}) = \int dn_0 \text{pr}(S_2, L | n_0, \mathcal{D}) \text{pr}(n_0 | \mathcal{D})$$

$$\text{pr}(n_0 | \mathcal{D}) \approx 0.17 \pm 0.01 \text{ fm}^{-3}$$

2 $\sigma$  ellipse (light yellow) is completely within the *conjectured* unitary gas limit

predicted range in  $S_v$  **agrees** with other **theoretical constraints**; but ~15 MeV stronger density-dependence of  $S_2(n_0)$

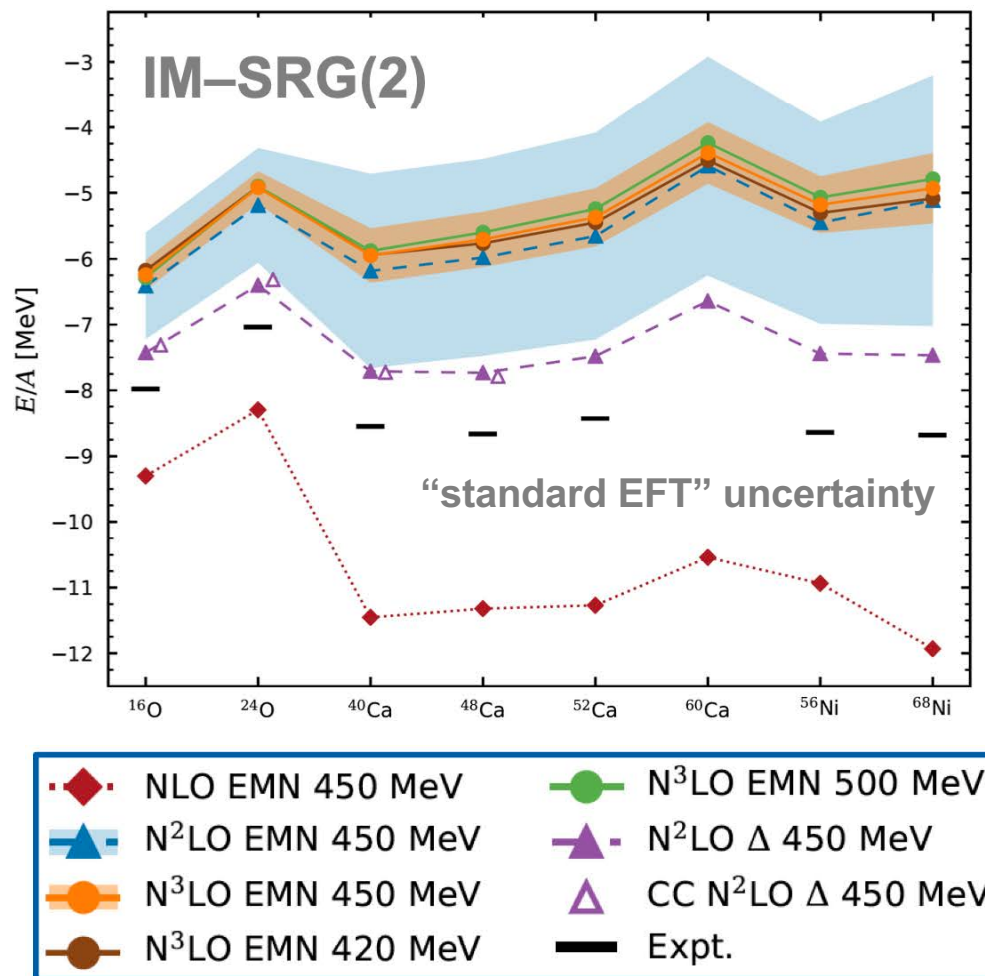
**two-dimensional Gaussian**

$$\begin{bmatrix} \mu_{S_v} \\ \mu_L \end{bmatrix} = \begin{bmatrix} 31.7 \\ 59.8 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 1.24 & 3.27 \\ 3.27 & 16.95 \end{bmatrix}$$

# Quantifying uncertainties in the nuclear-matter equation of state

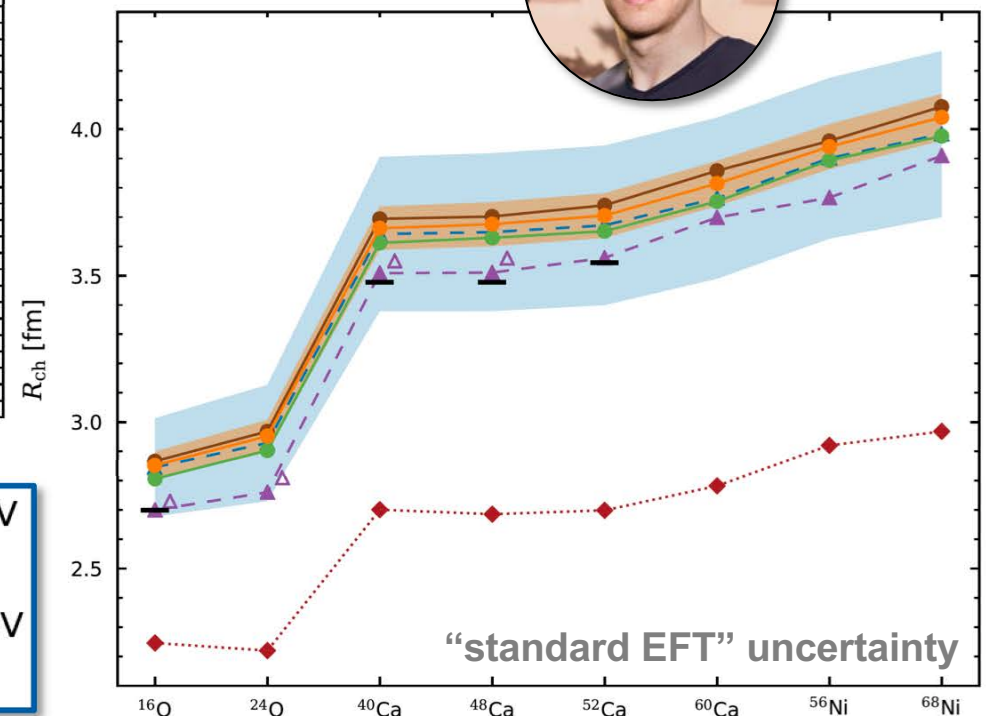
Results for nuclei with these potentials

## binding energies



realistic **saturation properties** may **not suffice** for medium-mass nuclei !

## charge radii



# Quantifying uncertainties in the nuclear-matter equation of state

## Summary and outlook

1

### set a new standard for UQ in infinite-matter calculations

- correlations within *and* between observables are crucial for reliable UQ
- need for *statistically* meaningful comparisons between theory and observation
- efficiently quantify and propagate EOS uncertainties to derived quantities

2

### *statistically rigorous* analysis of the EOS up to $N^3\text{LO}$

- excellent agreement of predicted  $S_V$ – $L$  correlation with experiment
- PNM and SNM show a regular EFT convergence pattern with increasing order
- extracted  $\Lambda_b$  is consistent with NN scattering •  $N^2\text{LO}$  coefficient may be an outlier

3

### improved NN+3N potentials up to $N^3\text{LO}$ are needed

- H  ther *et al.*, arXiv:1911.04955; Hoppe *et al.*, PRC **100**, 024318; ...

4

### full Bayesian UQ: MCMC for LECs & hyperparameters

- consistently include uncertainties in the LECs of chiral interactions
- compute nuclear saturation properties using Bayesian optimization

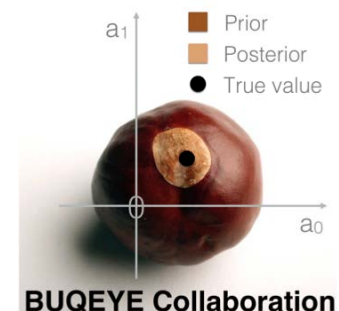
Unterst  tzt von / Supported by



Alexander von Humboldt  
Stiftung/Foundation

thanks to my collaborators:

|              |             |            |                |
|--------------|-------------|------------|----------------|
| R. Furnstahl | K. Hebeler  | J. Hoppe   | J. Melendez    |
| K. McElvain  | D. Phillips | A. Schwenk | C. Wellenhofer |

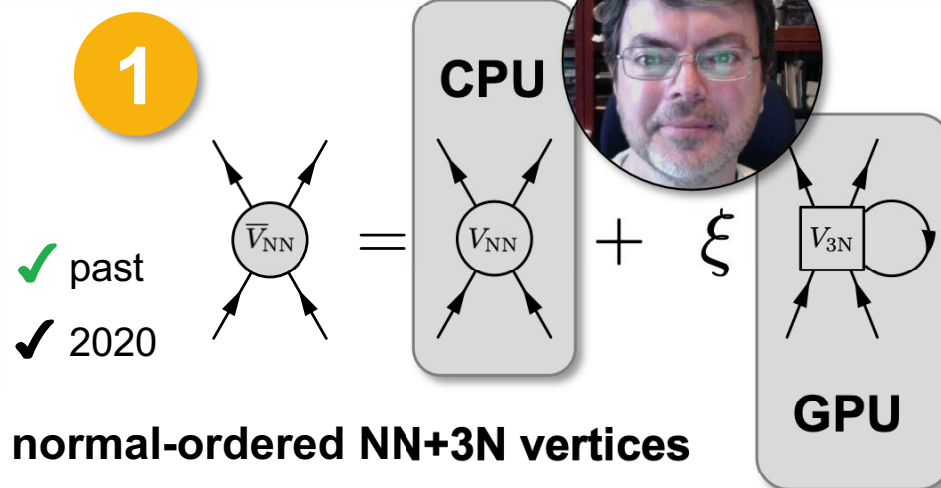


# Quantifying uncertainties in the nuclear-matter equation of state

## Next-generation MBPT calculations

| MBPT( $n$ )*  | 2    | 3     | 4     | 5      |
|---------------|------|-------|-------|--------|
| NN only       | ✓(1) | ✓(3)  | ✓(24) | ✓(375) |
| norm. ord. 3N | ✓    | ✓     | ✓     | ✓      |
| residual 3N   | ✓(1) | ✓(14) | —     | —      |

scalable multi-CPU/-GPU application



complete third order

## MC framework 2.0

SQL-based data repository

### asymmetric-matter EOS

self error detecting

openMP, MPI, and CUDA

MPI-JM

significantly improved accuracy

**highly optimized: > 100x faster**

3N partial-wave calculations



Automated generation and evaluation of many-body diagrams I. Bogoliubov many-body perturbation theory

Pierre Arthuis, Thomas Duguet, Alexander Tichai, Raphaël-David Lasserri, Jean-Paul Ebran  
Comput. Phys. **240**, 202

**2**