Truncation errors in effective field theory for infinite nuclear matter

Christian Drischler

Bayesian Inference in Subatomic Physics | September 17, 2019

"This […] **symposium will facilitate**

- **cross-communication** and **potential collaboration** on statistical applications among researchers from mathematics, statistics, and nuclear/particle physics […]
- try to **fill a knowledge gap** and provide a unique opportunity for physicists who are unfamiliar with Bayesian methods to start applying them to new problems"

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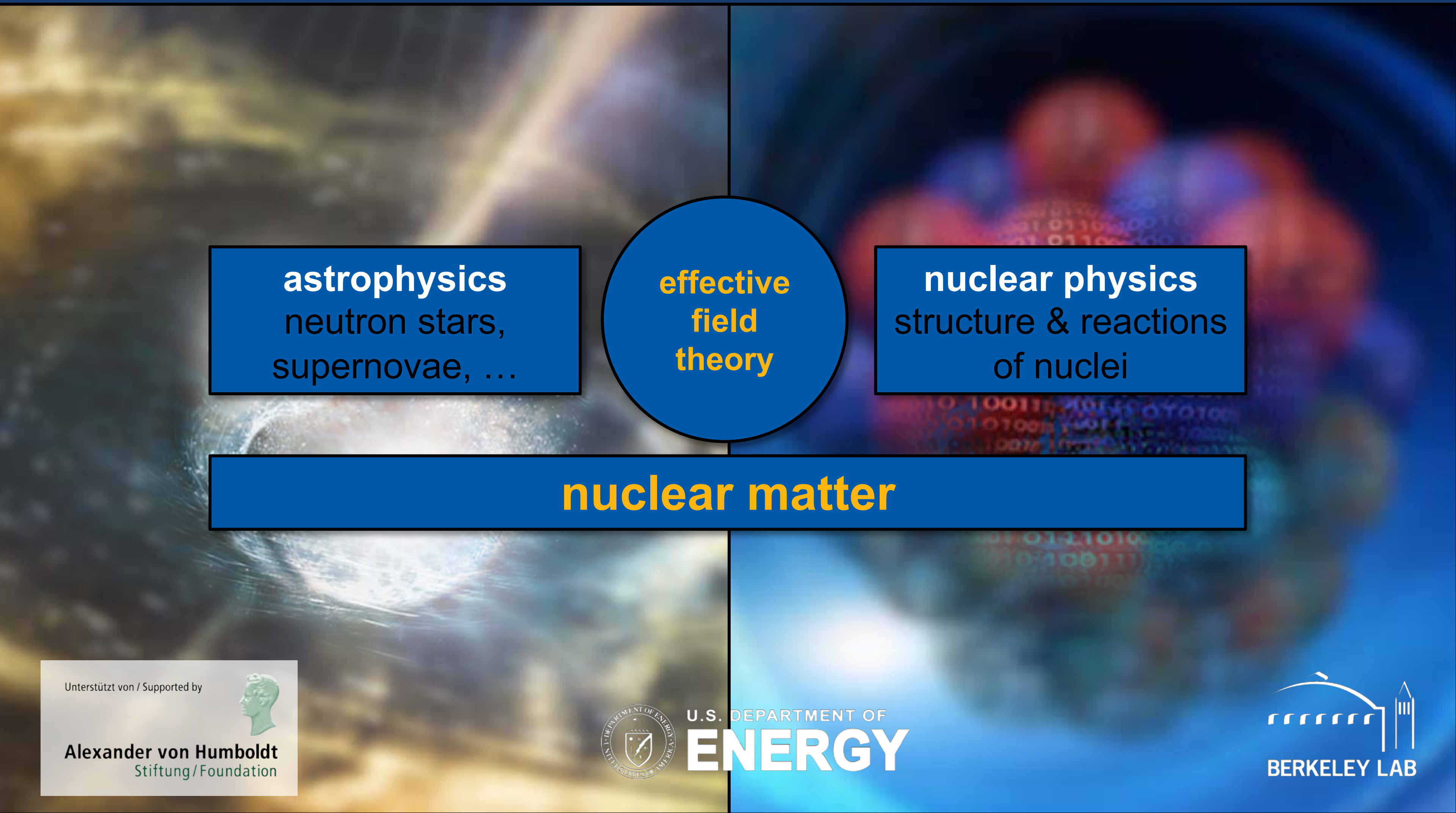

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(First) Direct detection of gravitational waves

ligo.caltech.edu

multi-messenger astronomy

+ Virgo
+ GEO600
+ …

Nobel Prize 2017

Binary Neutron-Star Merger

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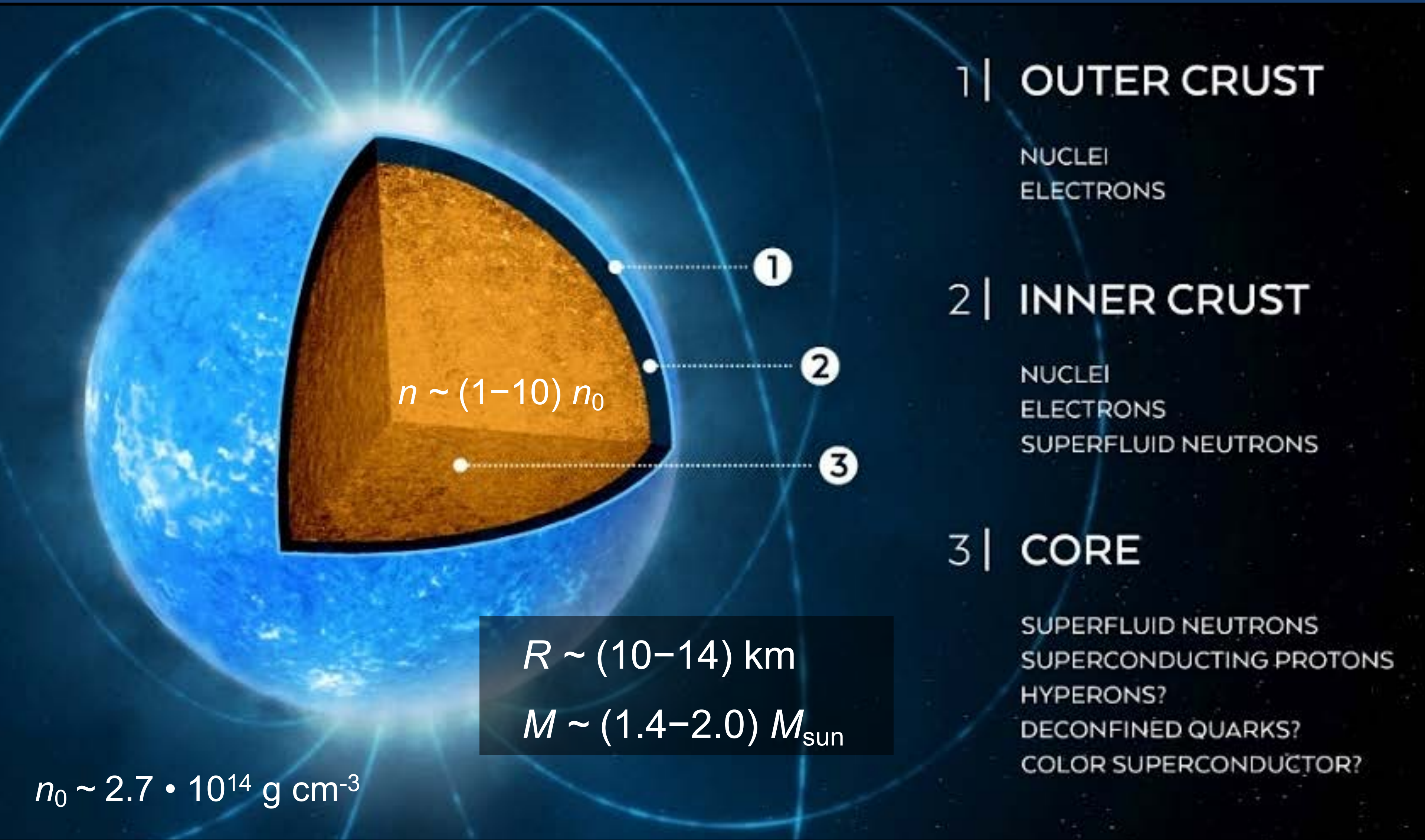
Neutron stars

e.g., Watts *et al.*, Rev. Mod. Phys. **88,** 021001



$n \sim (1\text{–}10)\, n_0$

$R \sim (10\text{–}14)$ km

$M \sim (1.4\text{–}2.0)\, M_{\rm sun}$

$n_0 \sim 2.7 \cdot 10^{14}$ g cm^{-3}

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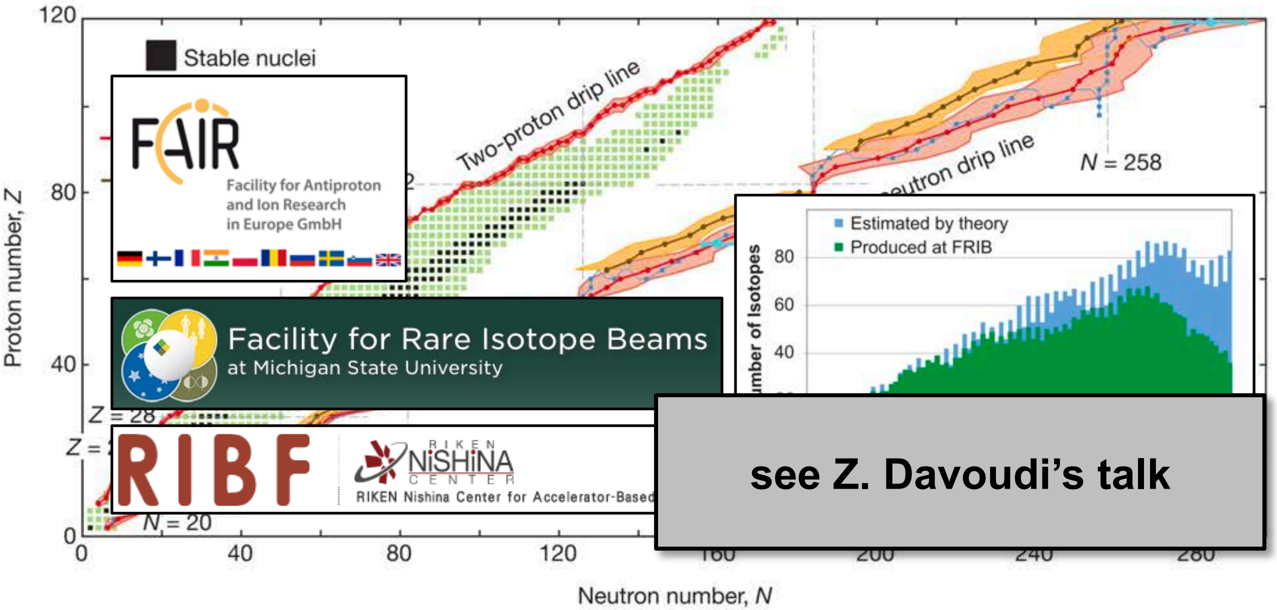
Erler *et al.*, Nature **486**, 509–512

Where do heavy elements come from?

How does the nuclear chart emerge from QCD?

How to predict properties of nuclei?



see Z. Davoudi's talk

observables

many-body framework

effective field theory

quantum chromodynamics

Truncation errors in effective field theory for infinite nuclear matter

see also Hebeler *et al.*, ARNPS **65**, 457

equation of state
neutron-star matter | nuclear saturation

many-body perturbation theory
computational efficient
many-body uncertainty estimates

chiral effective field theory
systematic expansion of nuclear forces
truncation error estimates

NPLQCD …

observables

many-body framework

effective field theory

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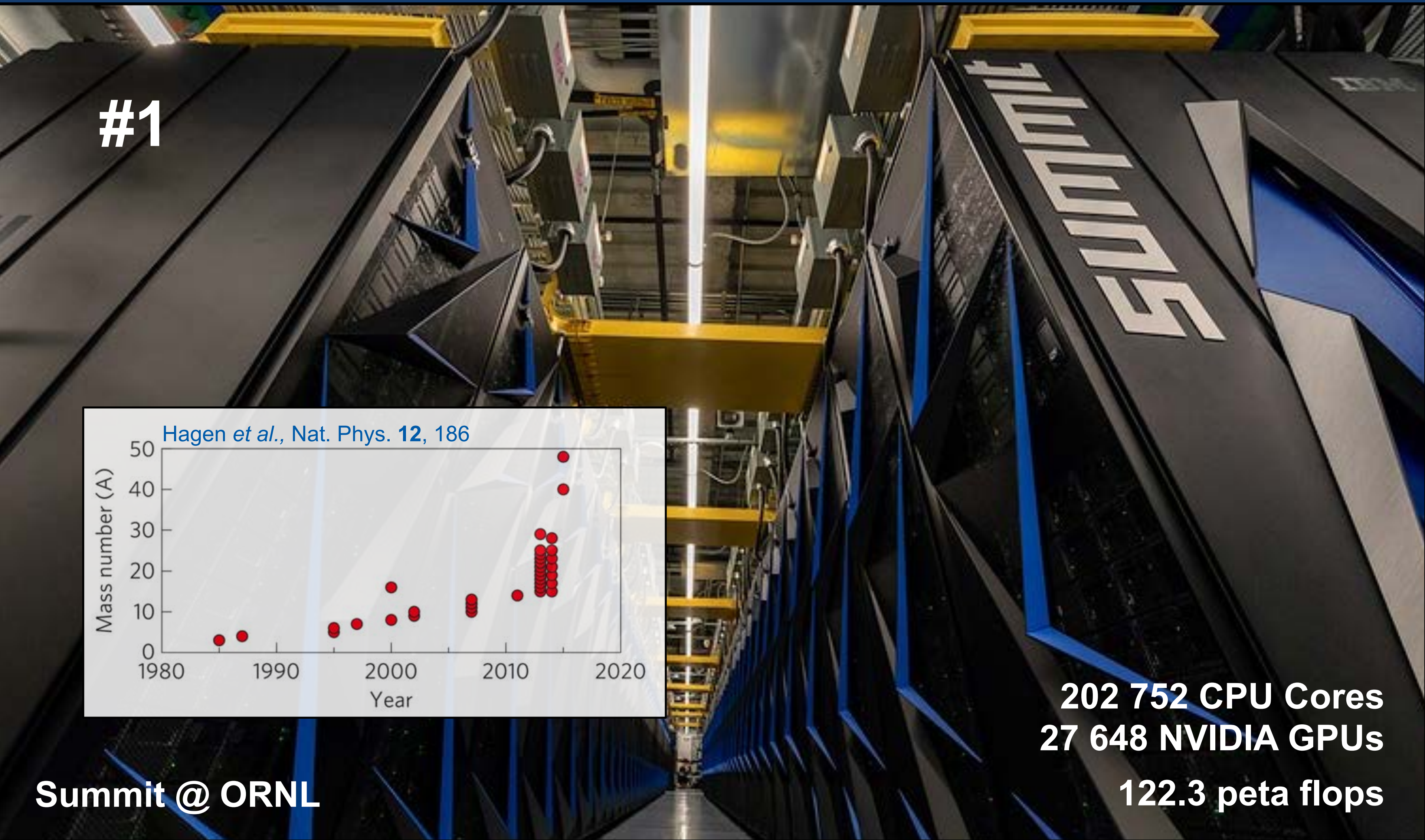
Next-generation supercomputers



#1

202 752 CPU Cores
27 648 NVIDIA GPUs
122.3 peta flops

Summit @ ORNL

Equation Of State

ground-state energy per particle of a system

$$\frac{E}{A}(n,\ \beta,\ T)$$

total density
neutron excess
temperature

consisting of neutron and protons

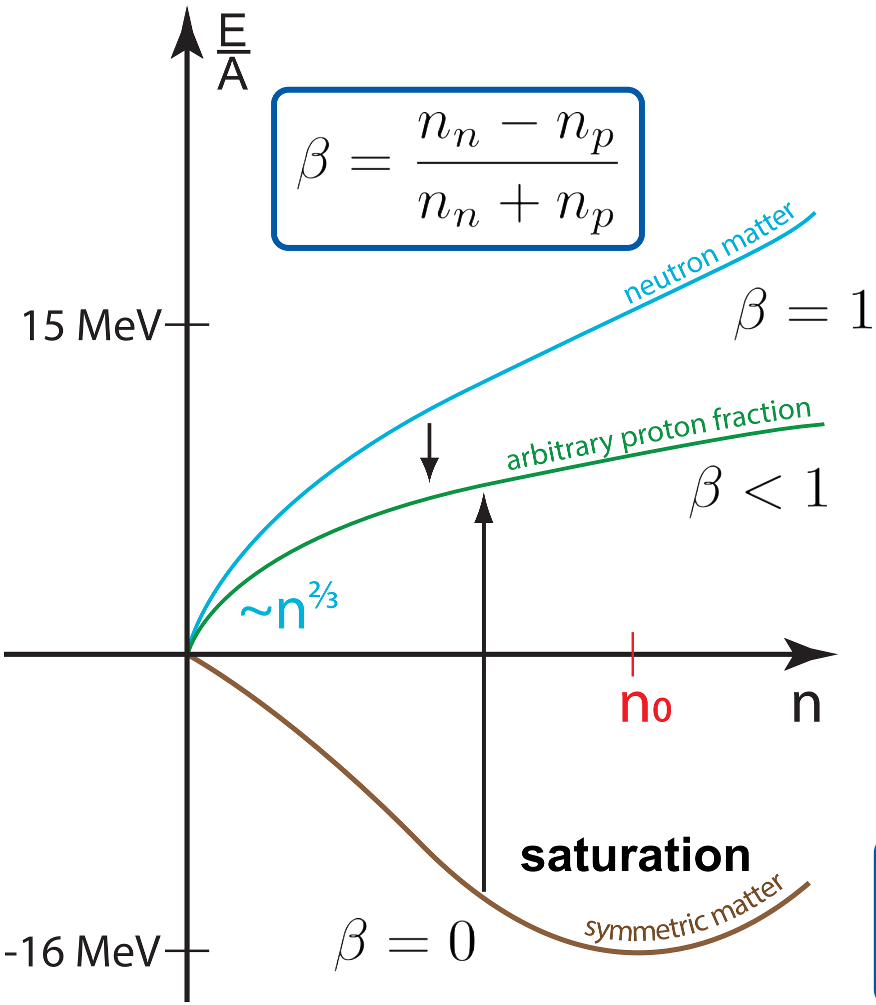
$$n = n_n + n_p$$

neutron | proton density

$$\beta = \frac{n_n - n_p}{n}$$

Truncation errors in effective field theory for infinite nuclear matter

Homogeneous nuclear matter



theoretical **testbed** for nuclear forces with important consequences for EOS

saturation point
($n_0 \sim 0.16\ \mathrm{fm}^{-3}$, $a_v \sim 16$ MeV)

incompressibility ($K \sim 240$ MeV)

symmetry energy ($E_{sym} \sim 32$ MeV) and its **slope** ($L \sim 55$ MeV) at n_0

$$\boxed{11.1\,\mathrm{km} \leqslant R_{1.4\,\mathrm{M}_\odot} \leqslant 12.7\,\mathrm{km}}$$

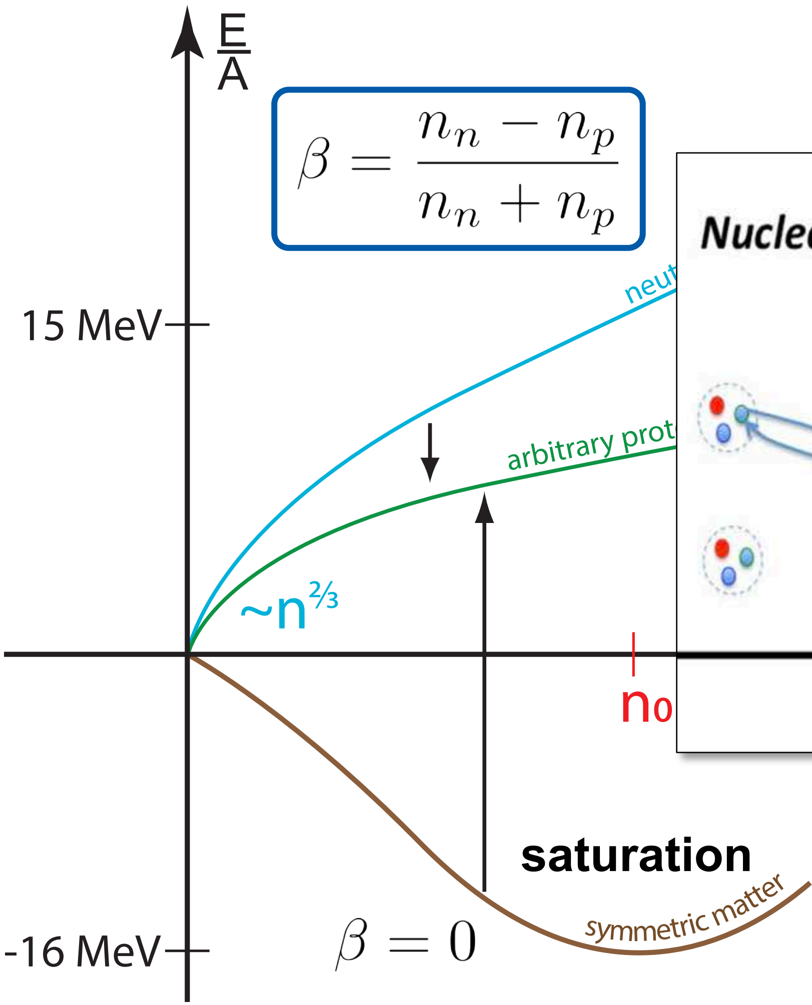
Hagen *et al.*, Nat. Phys. **12**, 186

$$\boxed{\frac{E}{A}(\beta, n) = \frac{E}{A}(\beta = 0, n) + \beta^2 E_{\mathrm{sym}}(n)}$$

Truncation errors in effective field theory for infinite nuclear matter

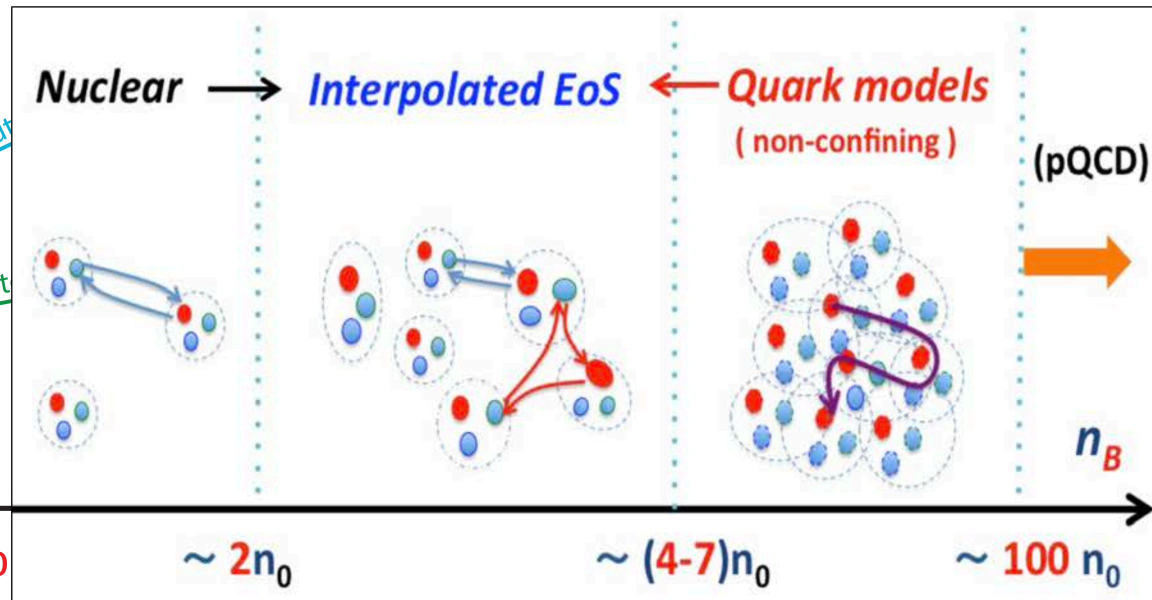
Homogeneous nuclear matter

theoretical **testbed** for nuclear forces with important consequences for EOS



$$\beta = \frac{n_n - n_p}{n_n + n_p}$$

$$\beta = 0$$



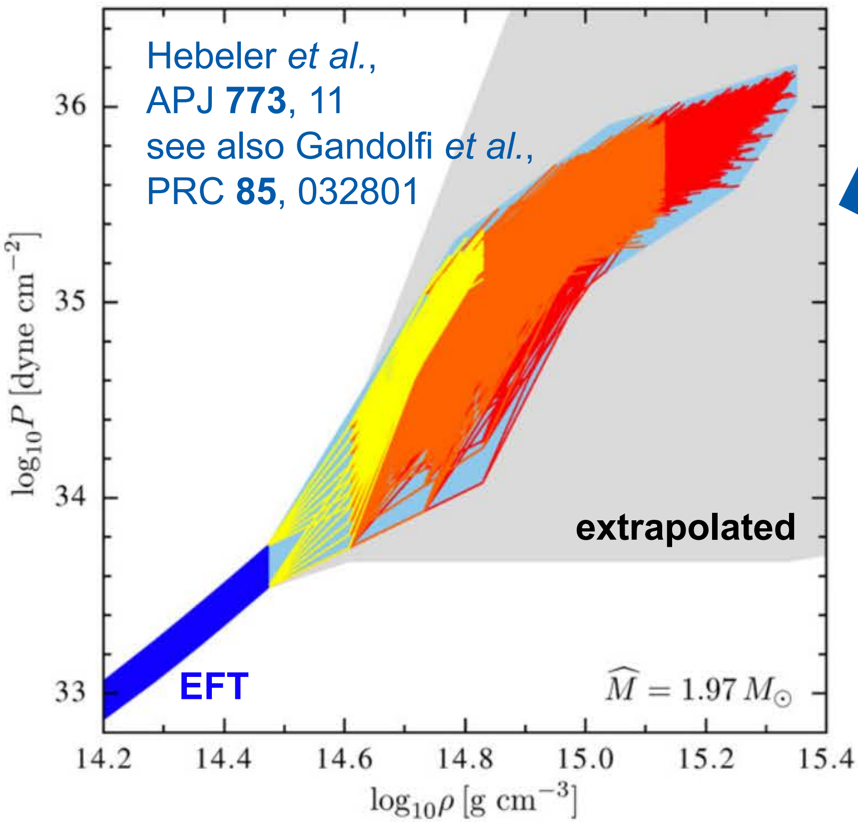
Hagen *et al.*, Nat. Phys. **12**, 186

$$\frac{E}{A}(\beta, n) = \frac{E}{A}(\beta = 0, n) + \beta^2 E_{\mathrm{sym}}(n)$$

Truncation errors in effective field theory for infinite nuclear matter

Mass–radius relation

see, *e.g.*, Greif *et al.*, MNRAS **485**, 4



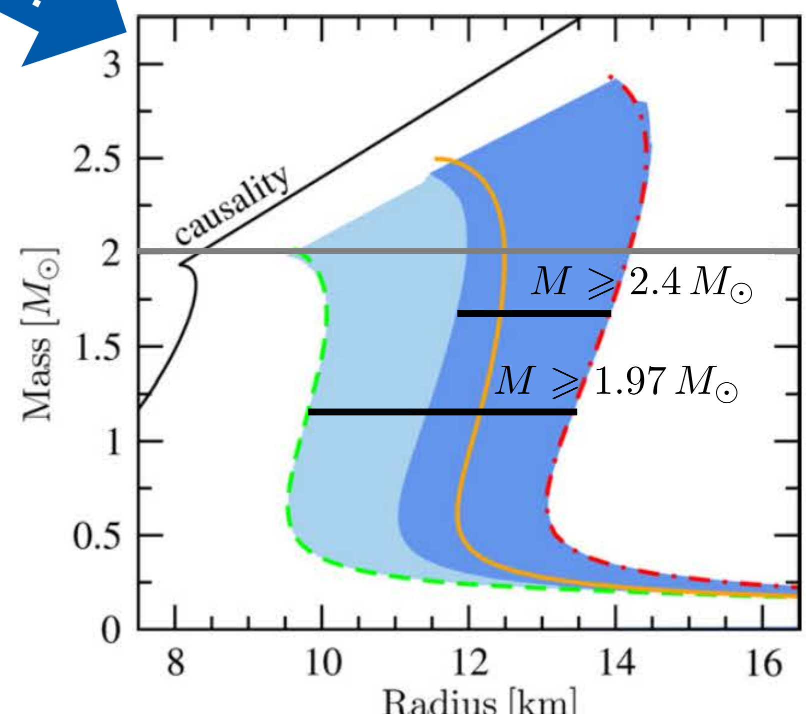
$$R_{1.4\,M_\odot} = 9.7 - 13.9\,\mathrm{km}$$

TOV eqs.

constraints from

- \+ causality
- \+ $M_{max} \geq 1.97\ M_{sun}$
- \+ GW170817, …

mass–radius relation



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Mass–radius relation

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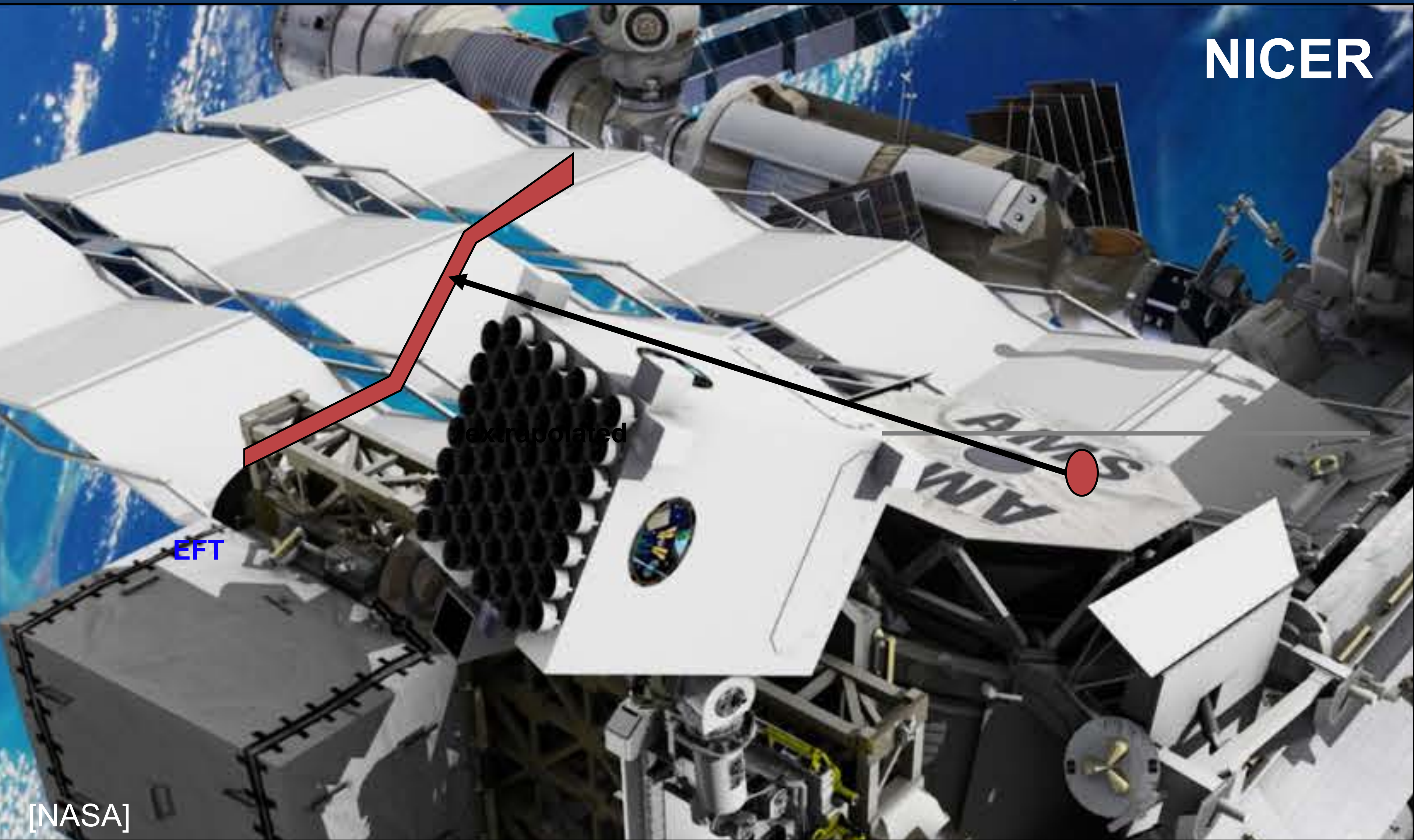

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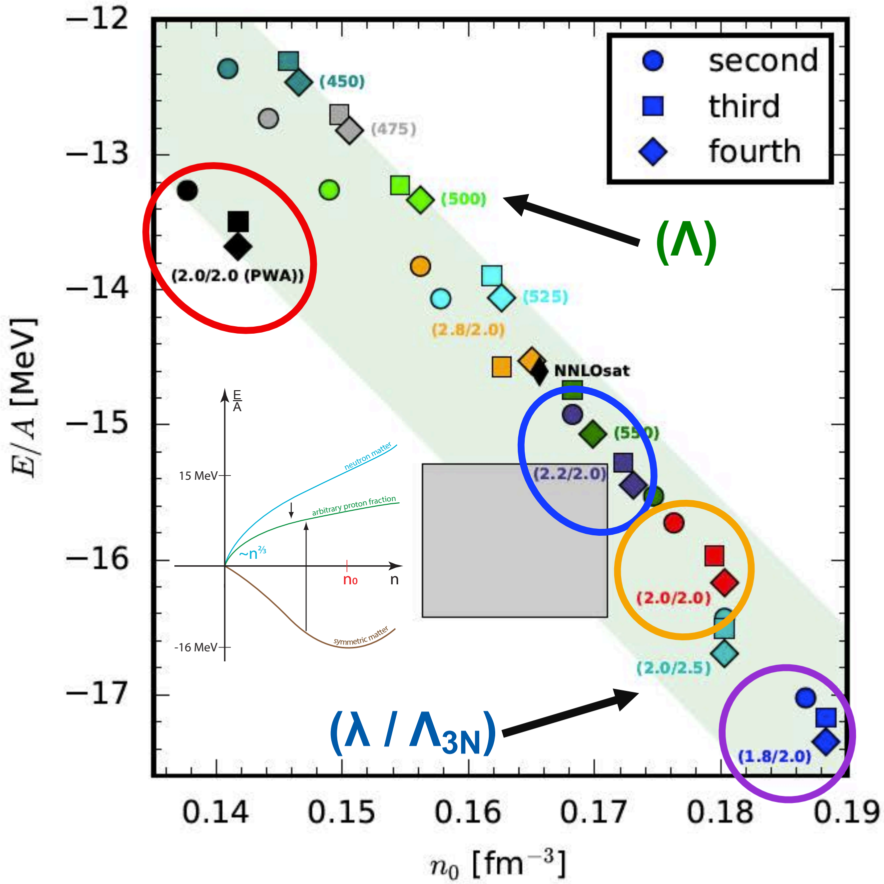
Nuclear saturation

see also Hoppe, CD, Hebeler *et al.*, PRC **100**, 024318

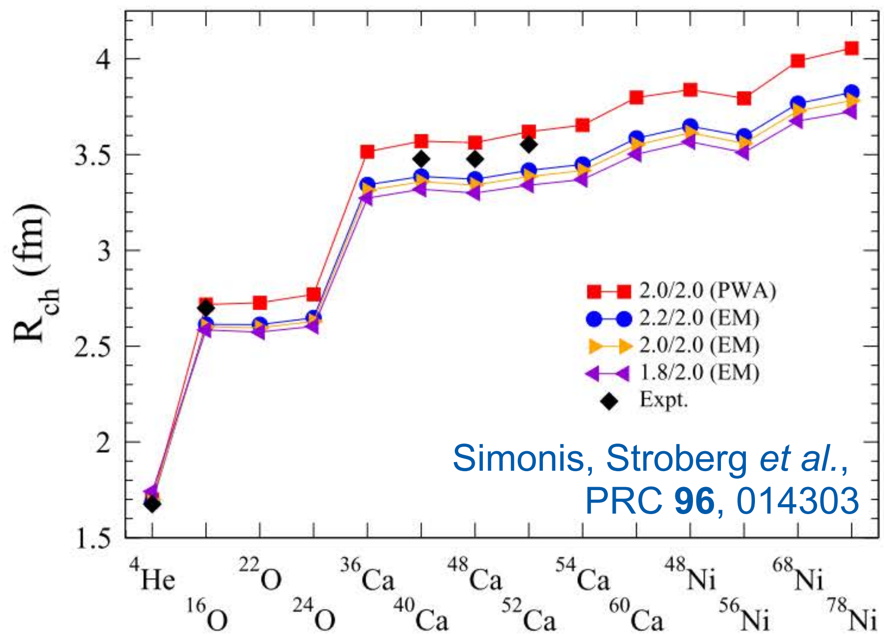
Homogeneous Matter



CD, Hebeler, Schwenk, PRL **122**, 042501

magic 1.8 / 2.0 (EM) agrees well with experimental data!

Finite Nuclei

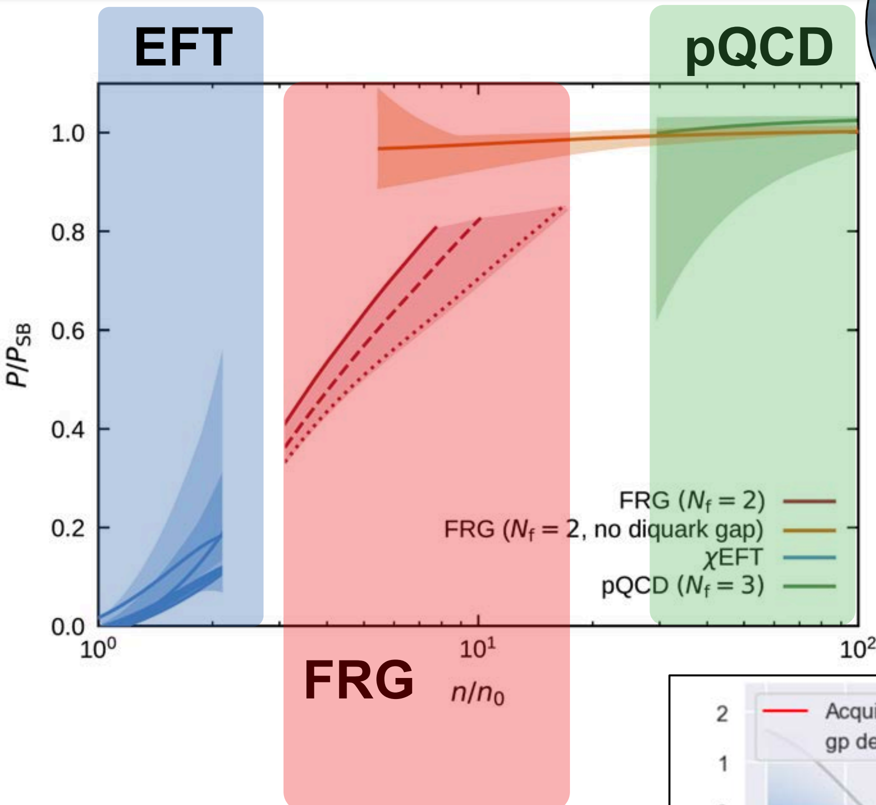


Simonis, Stroberg *et al.*, PRC **96**, 014303

Truncation errors in effective field theory for infinite nuclear matter

Derived quantities: pressure

Leonhardt, Pospiech, Schallmo, Braun, CD, Hebeler, Schwenk,
arXiv:1907.05814

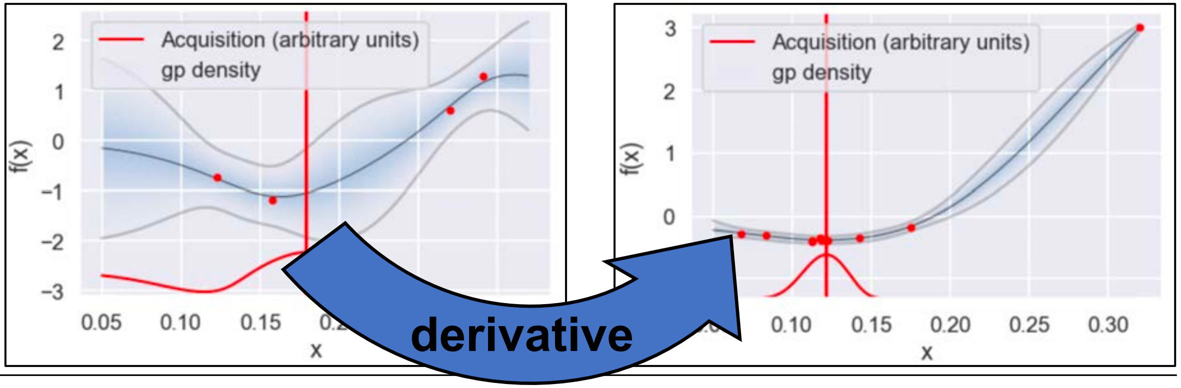


Pressure

$$P(n,\, \beta) = n^2 \frac{\partial E/A}{\partial n}(n,\, \beta)$$

EFT seems to match first constraints from QCD at intermediate densities

in future, derivatives using GPs instead of finite differencing?



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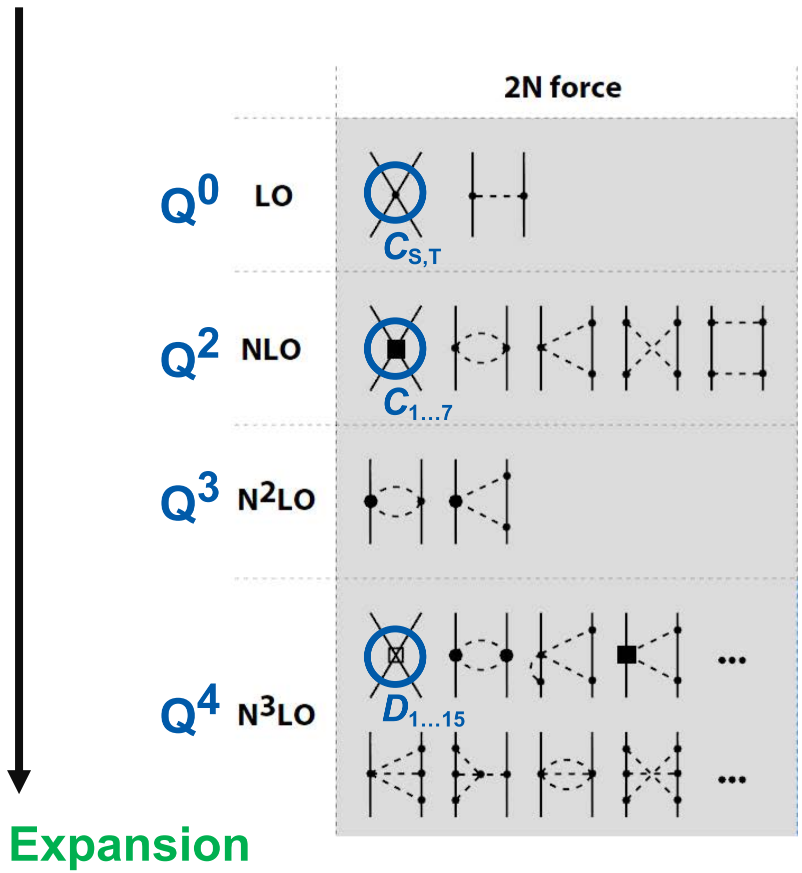
Hierarchy of nuclear forces in chiral EFT

e.g., Machleidt, Entem, Phys. Rep. **503**, 1



modern approach to nuclear forces:

- QCD is nonperturbative at the low-energy scales of nuclear physics
- use relevant instead of the fundamental degrees of freedom: *e.g.*, **nucleons** and **pions**
- **pion exchanges** and short-range **contact interactions** ($\propto$ LEC)
- systematic expansion enables improvable **uncertainty estimates**

$$\boxed{Q = \max\left(\frac{p}{\Lambda_b}, \frac{m_\pi}{\Lambda_b}\right) \sim \frac{1}{3}}$$

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Krebs, Machleidt, Meißner, ...

Truncation errors in effective field theory for infinite nuclear matter

Hierarchy of nuclear forces in chiral EFT

e.g., Machleidt, Entem, Phys. Rep. **503**, 1

Many-body forces

		2N force	3N force	4N force
Q^0	LO			
Q^2	NLO	$C_{1\ldots7}$		
Q^3	N^{2}LO			
Q^4	N^{3}LO	$D_{1\ldots15}$	no unknown parameters	

$$\chi_k = \chi_{\text{ref}} \sum_{n=0}^{k} c_n Q^n$$

see also J. Melendez's and D. Phillips's talk

$$V = \sum_{n=0}^{\infty} \left(C_i^{(n)} V^{(n)} \right) Q^n$$

S.Weinberg

Expansion

… and ongoing work at N^{4}LO and even N^{5}LO…

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Krebs, Machleidt, Meißner, ...

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Many new potentials available!

Semilocal momentum-space regularized chiral two-nucleon potentials up to fifth order

P. Reinert,[1,*] H. Krebs,[1,†]

[1]Institut für Theoretische Physik II, Ruhr-Univers

High-quality two-nucleon potentials up to fifth order of the chiral expansion

D. R. Entem,[1,*] R. Machleidt,[2,†] and Y. Nosyk[2]

[1]Grupo de Física Nuclear, IUFFyM, Universidad de Salamanca, E-37008 Salamanca, Spain

o 83844, USA

Uncertainty Analysis and Order-by-Order Optimization of Chiral Nuclear Interactions

B. D. Carlsson,[1,*] A. Ekström,[2,3,†] C. Forssén,[1,2,3,‡] D. Fahlin Strömberg,[1] G. R. Jansen,[3,4]

[1]Depar
[2]Departme
[3]Phy

Minimally nonlocal nucleon-nucleon potentials with chiral two-pion exchange including Δ resonances

M. Piarulli,[1] L. Girlanda,[2,3] R. Schiavilla,[1,4] R. Navarro

see M. Piarulli's talk

6, USA

Computacional Universidad de Granada,

Δ isobars and nuclear saturation

A. Ekström,[1] G. Hagen,[2,3] T. D. Morris,[2,3] T. Papenbrock,[2,3] and P. D. Schwartz[2,3]

[1]Department of Physics,
[2]Physics Division, Oak
[3]Department of Physics and

Three-nucleon force in chiral EFT with explicit $\Delta(1232)$ degrees of freedom: Longest-range contributions at fourth order

H. Krebs,[1,*] A. M. Gasparyan,[1,2,†] and E. Epelbaum[1,‡]

[1]Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany

[2]Institute for Theoretical and Experimental Physics, B. Cheremushkinskaya 25, 117218 Moscow, Russia

e.g., Carlsson, Ekström, Entem, Epelbaum, Forssén, Gezerlis, Krebs, Machleidt, Piarulli, Reinert, Tews

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Efficient Monte Carlo framework

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CD, Hebeler, Schwenk, PRL **122**, 042501

efficient evaluation of **diagrams**
(single-particle basis)

- **implementing diagrams** has become **straightforward** (also particle-hole or 3N terms)
- using **automatic code generation** based on analytic expressions
- multi-dimensional momentum integrals: VEGAS (openMP, MPI, and CUDA)
- **computational beast:** controlled computation of arbitrary interaction or many-body diagrams

EOS up to high orders

automatic code generation

analytic form of the diagrams

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Number of diagrams in MBPT

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Stevenson, Int. J. Mod. Phys. C **14**, 1135

The number of diagrams increases rapidly!

	1,	**3,**	**39,**	**840,**	**27 300,**	**1 232 280,**	**…**
n =	2	3	4	5	6	7	

Integer sequence A064732:
Number of labeled Hugenholtz diagrams with n nodes.

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Number of diagrams in MBPT

Stevenson, Int. J. Mod. Phys. C **14**, 1135

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see also Hebeler *et al.*, ARNPS **65**, 457

equation of state
neutron-star matter | nuclear saturation

many-body perturbation theory
computational efficient
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chiral effective field theory
systematic expansion of nuclear forces
truncation error estimates

CalLat NPLQCD • • •

observables

many-body framework

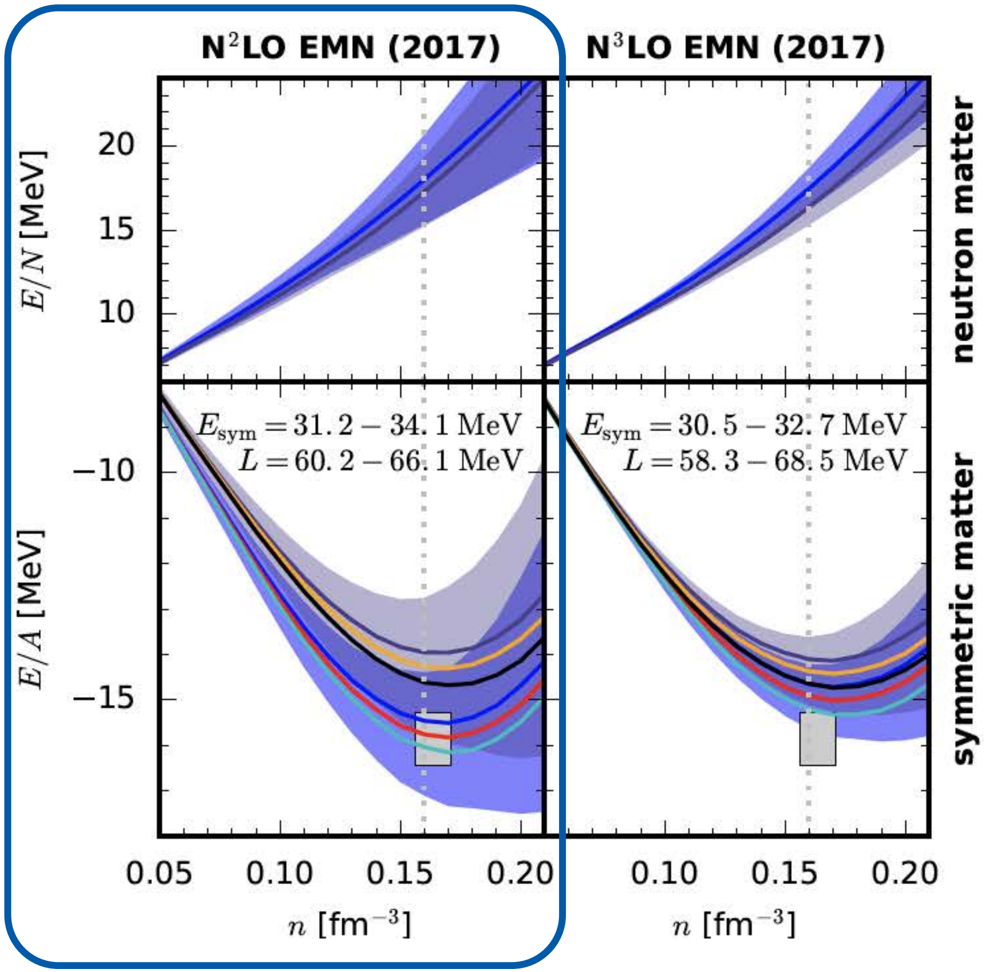
effective field theory

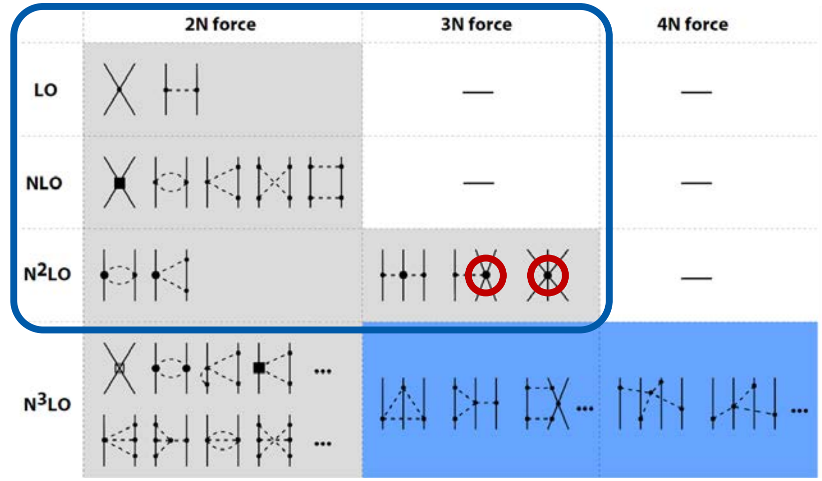
quantum chromodynamics

Truncation errors in effective field theory for infinite nuclear matter

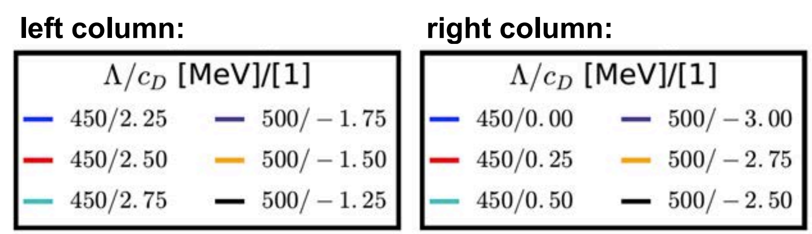
Neutron and nuclear matter at N^{3}LO

CD, Hebeler, Schwenk, PRL **122**, 042501





reduced uncertainties due to N^{3}LO contributions !



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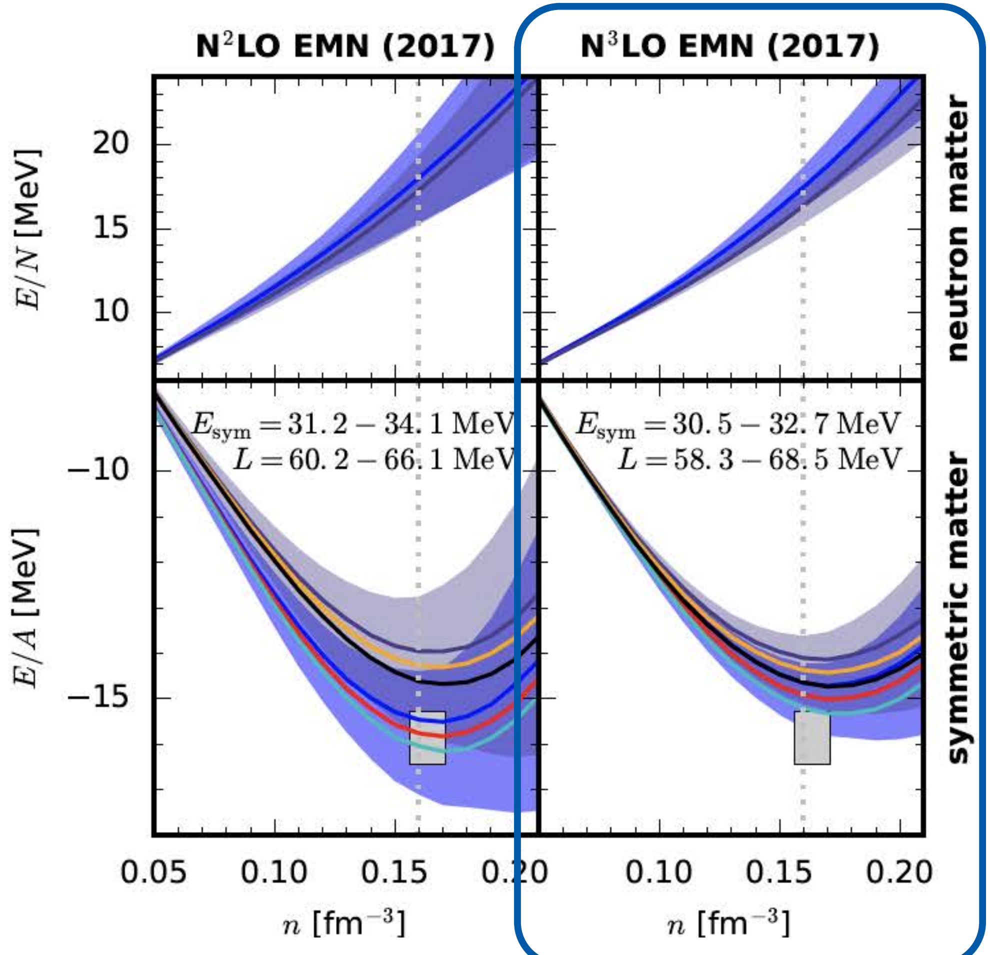
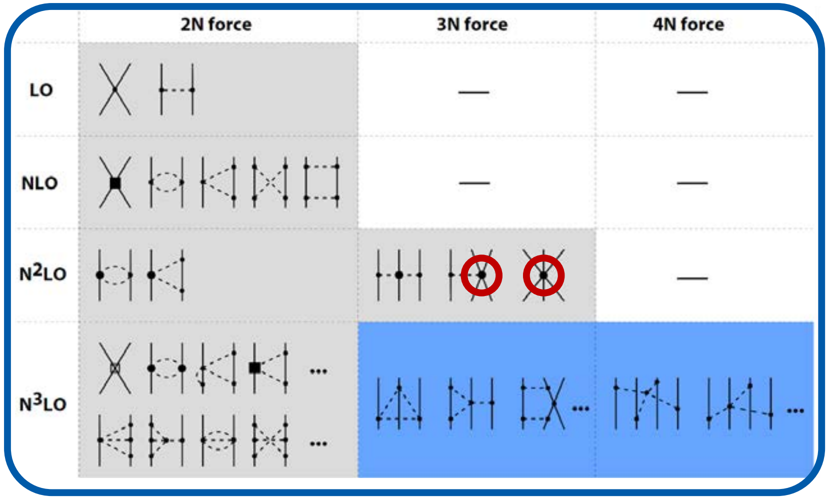
Neutron and nuclear matter at N^{3}LO

CD, Hebeler, Schwenk, PRL **122**, 042501





reduced uncertainties due to N^{3}LO contributions !

left column:

Λ/c_D [MeV]/[1]	
450/2.25	500/−1.75
450/2.50	500/−1.50
450/2.75	500/−1.25

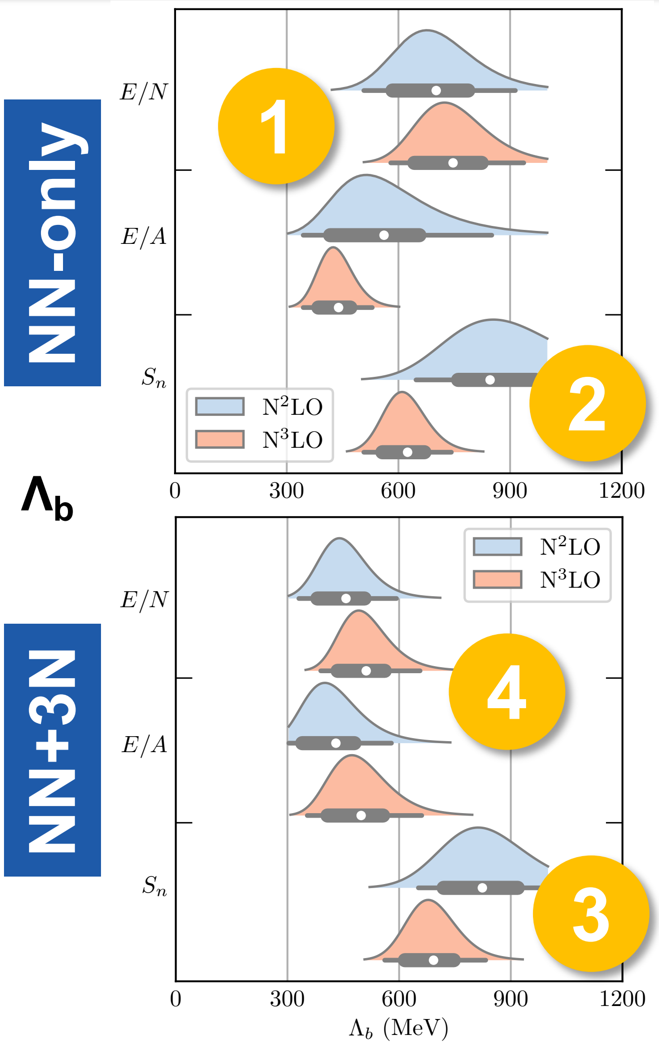
right column:

Λ/c_D [MeV]/[1]	
450/0.00	500/−3.00
450/0.25	500/−2.75
450/0.50	500/−2.50

Truncation errors in effective field theory for infinite nuclear matter

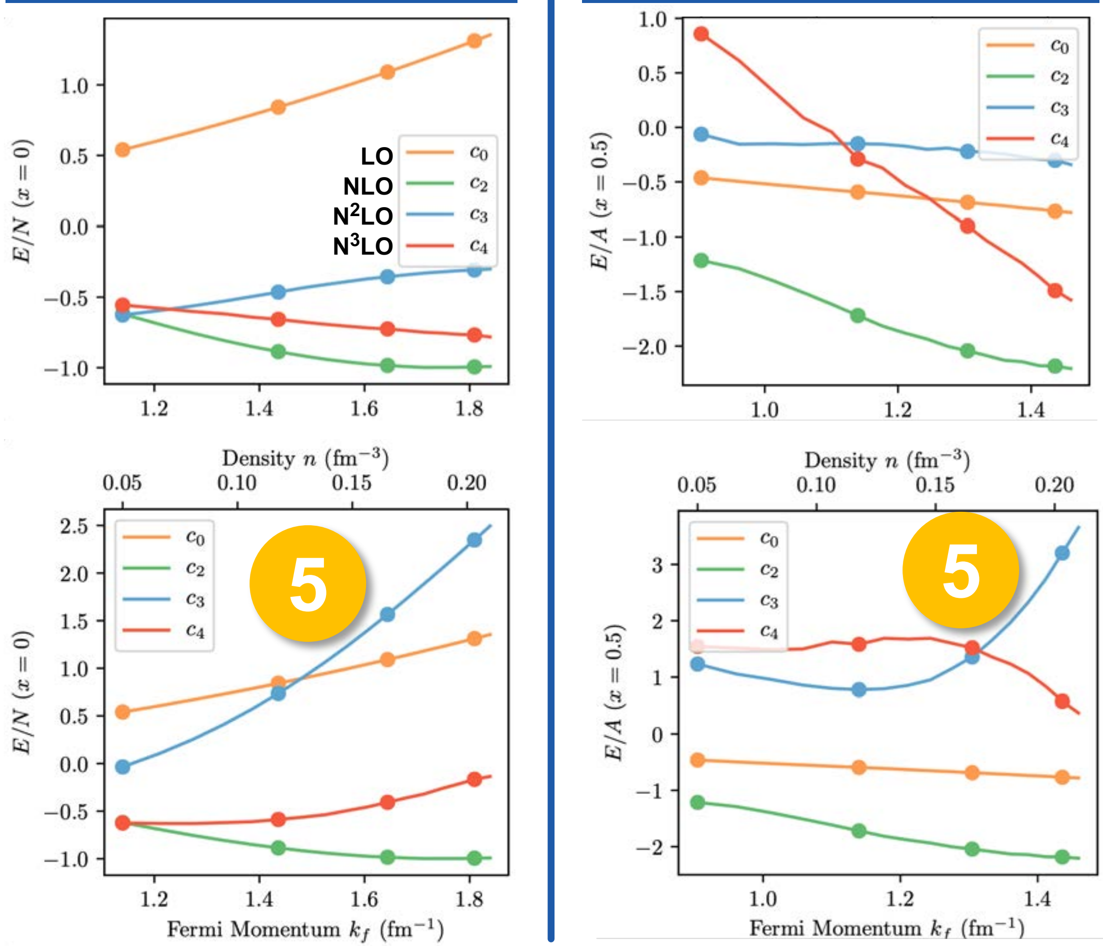
Truncation error analysis: $\Lambda = 450$ MeV

with Melendez, Furnstahl, Phillips



Neutron matter

Symmetric matter



Truncation errors in effective field theory for infinite nuclear matter

Truncation error analysis: Λ = 450 MeV

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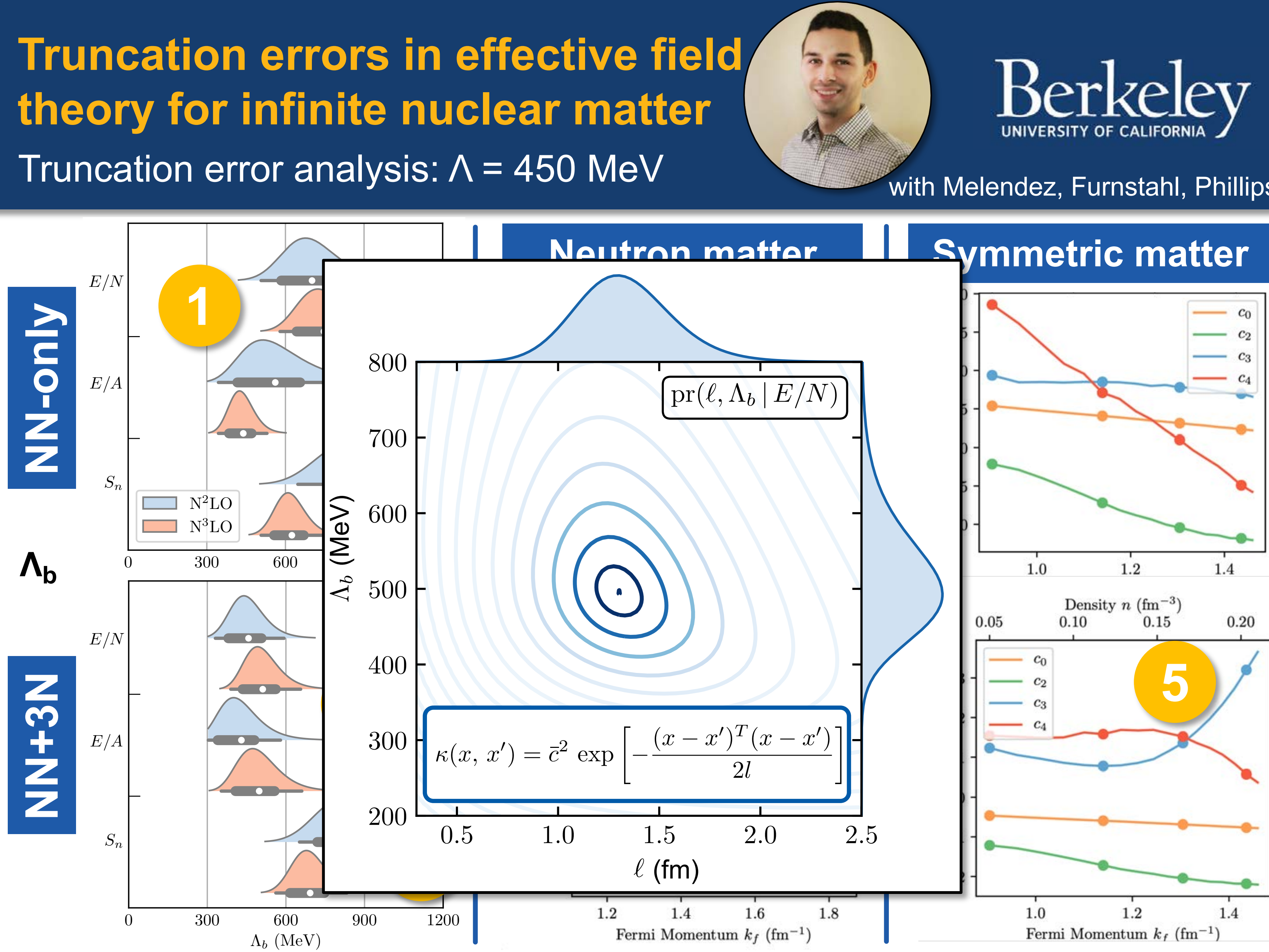

Truncation errors in effective field theory for infinite nuclear matter

Questions for discussion(s) I

What is the physical interpretation of the **correlation length** in nuclear matter? Is there a 1:1 mapping from the incompressibility to that correlation length? How could we make use of that information?

Our analysis infers a most probable **expansion parameter**. **Should** one then assume a momentum scale (*e.g.*, k_F) to convert this to a breakdown scale? Or, should one assume a breakdown scale (*e.g.*, Λ_b) from an NN analysis and convert the expansion parameter to a momentum scale in infinite matter?

How should uncertainties from the EOS be propagated to derived quantities, *e.g.*, pressure or speed of sound? Will the use of GPs as interpolants for the EFT coefficients mean that it is easy to reconstruct such quantities which are computed as **derivatives of the EOS**?

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Questions for discussion(s) II

What is the 2D 68% confidence region on the saturation point? How does this change if that region is conditioned not just on EDFs, but also on information from *ab initio* calculations?

What would it take to include data on the empirical saturation point, or constraints on the neutron-matter EOS from neutron-star observations, in fits of nuclear forces from chiral EFT?

How should we score different chiral EFT forces against such data? How many orders are enough given the current accuracy of *experimental* constraints on infinite matter? What degrees of freedom, *e.g.*, delta-full vs. chiral EFT, do we need to consider?

How should we deal with soft potentials that cause suppressed contributions from odd chiral orders? Separate expansions for even | odd orders? What are the implications for truncation errors?

Truncation errors in effective field theory for infinite nuclear matter

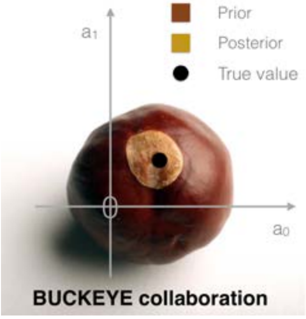
Summary and outlook

1. **Perform zero-T calculations (up to high order)**
   resummation, higher-order single-particle spectra, …
2. **Work out finite-T extension (to third order)**
   finish developments, study thermodynamic properties, …
3. **Construct high-density | temperature EOS**
   observational constraints, interface to astrophysics, …
4. **Quantify theoretical uncertainties**
   Bayesian truncation errors: naturalness, breakdown scale, …

Collaborators:

R. Furnstahl
J. Melendez
D. Phillips

K. Hebeler
K. McElvain
A. Schwenk



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