

Eigenvector Continuation for scattering with local chiral NN and optical potentials

MICHIGAN STATE
UNIVERSITY

Christian Drischler

April 26, 2021 | INT 21-1b: Nuclear Forces for Precision Nuclear Physics

This week's topic:

Improving nuclear forces with *novel fitting strategies* and higher orders in chiral EFT

1

using EC as an efficient emulator for scattering with local chiral interactions and optical potentials

2

statistical quantification and propagation of EFT truncation errors in nuclear matter calculations

Keywords:

- + ChEFT + scattering
- + Variational principles
- + Eigenvector Continuation
- + Bayesian UQ
- + infinite nuclear matter
- + EFT truncation errors
- + ...

- + LVC
- + Virgo
- + GEO600
- + KAGRA
- + ...



What is the secondary object
in GW190425 and GW190814 ?

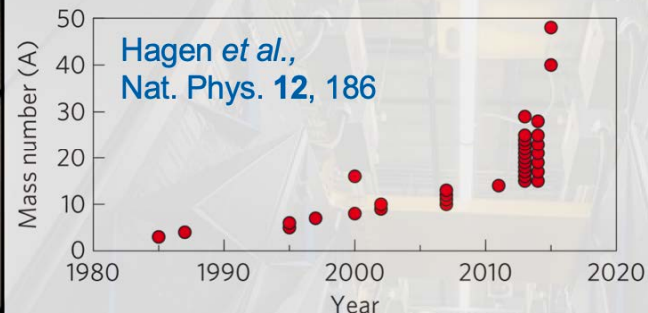
NICER

- + STROBE-X
- + eXTP
- + ...



Facility for Rare Isotope Beams
at Michigan State University

- + FRIB
- + RHIC
- + LHC
- + FAIR
- + ...



See Jeremy Holt's talk
*Normalizing flows for microscopic
calculations of the nuclear EOS*

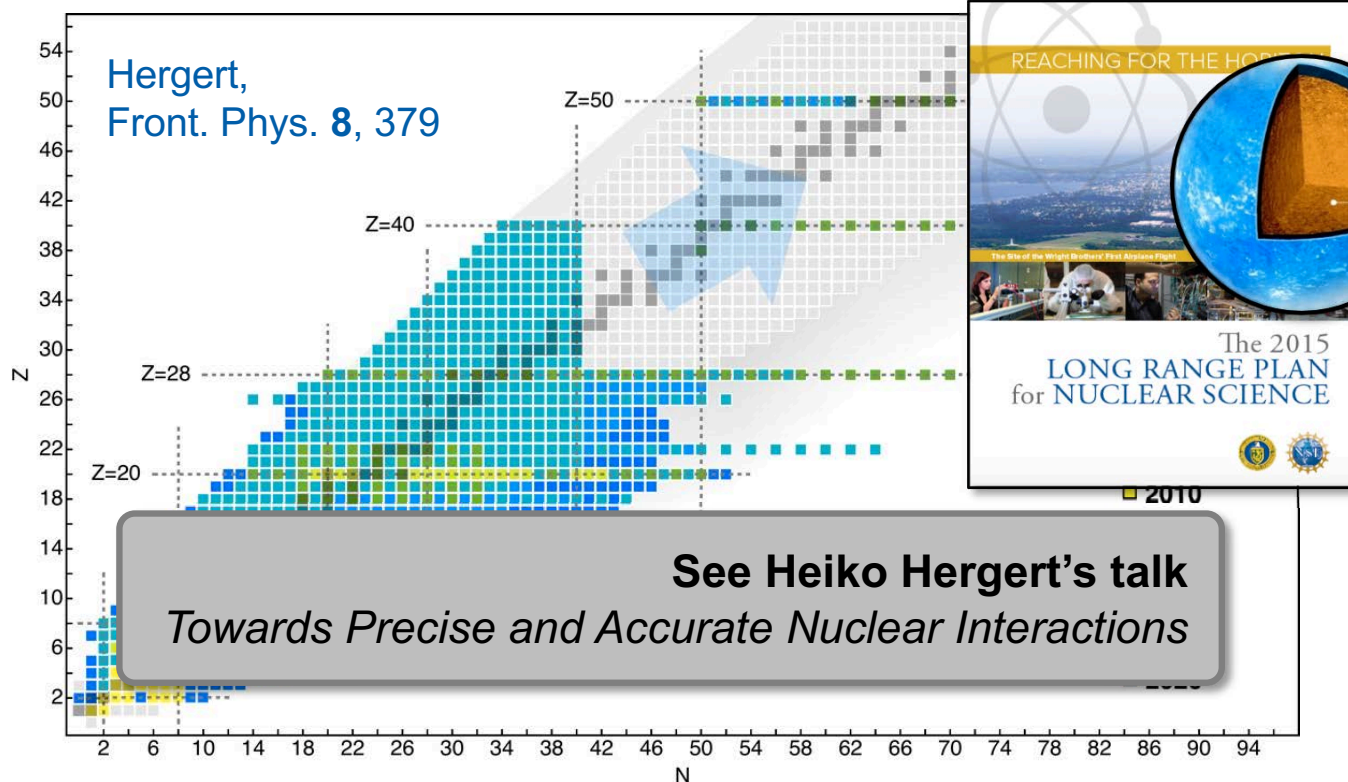
Eigenvector Continuation for scattering with local chiral NN and optical potentials

CD, Haxton, McElvain *et al.*, arXiv:1910.07961 (PPNP in press)

How does the nuclear chart emerge from QCD?

Where do heavy elements come from?

How does subatomic matter organize itself?



observables

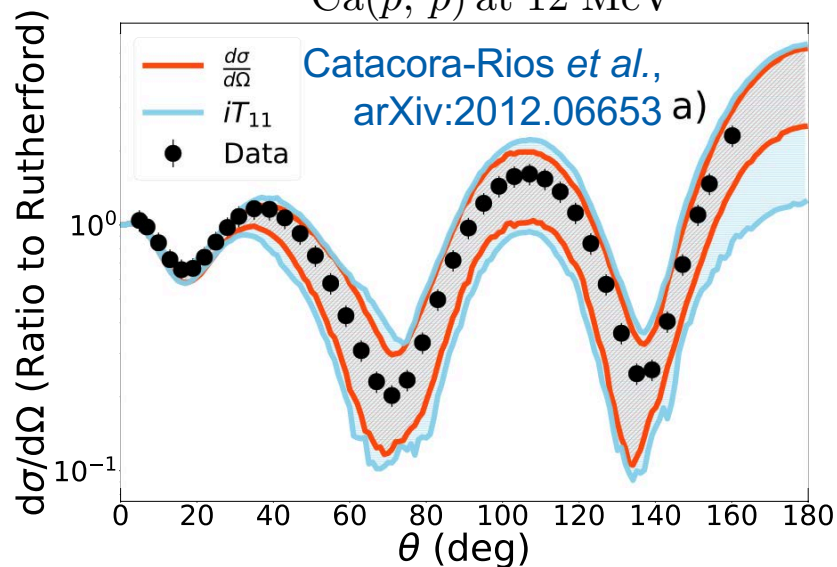
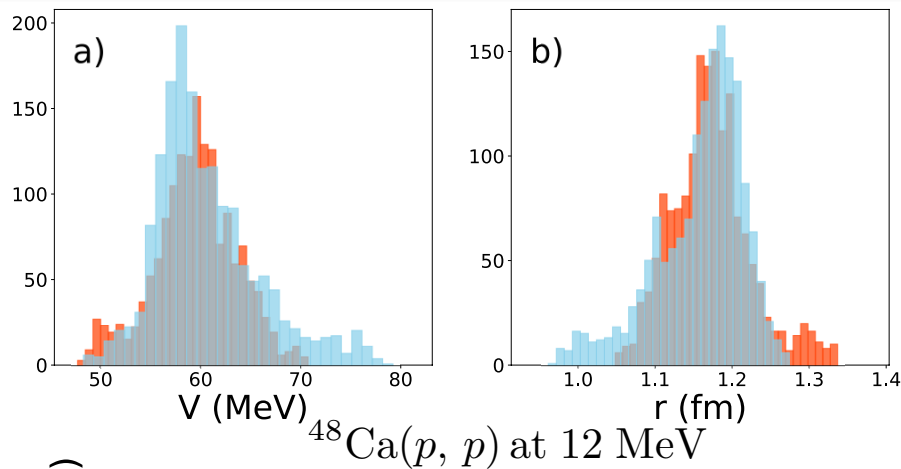
many-body framework

effective field theory

quantum chromodynamics

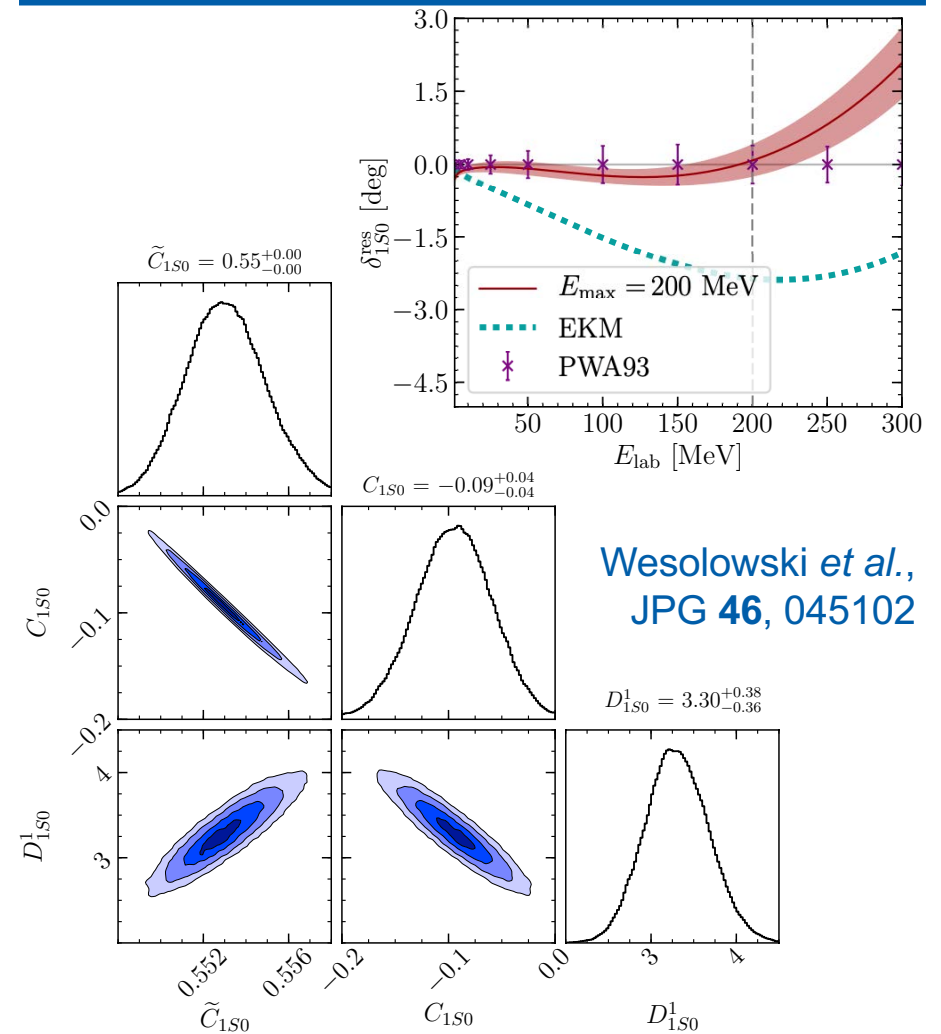
Eigenvector Continuation for scattering with local chiral NN and optical potentials

Bayesian uncertainty quantification



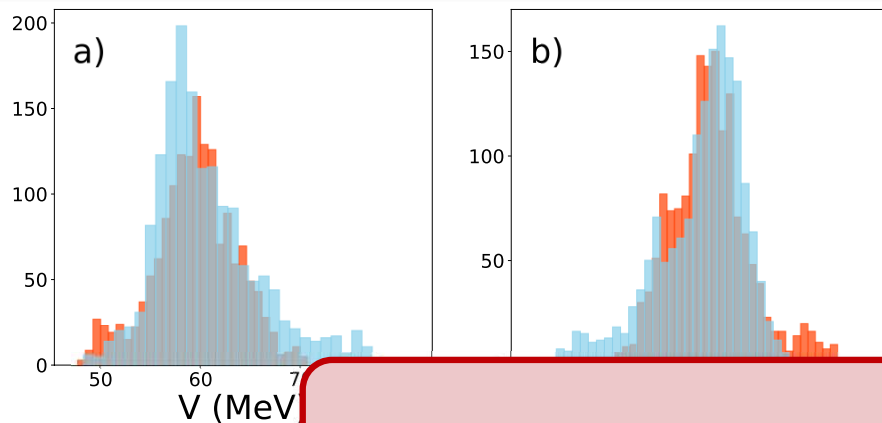
Optical potentials

Chiral potentials

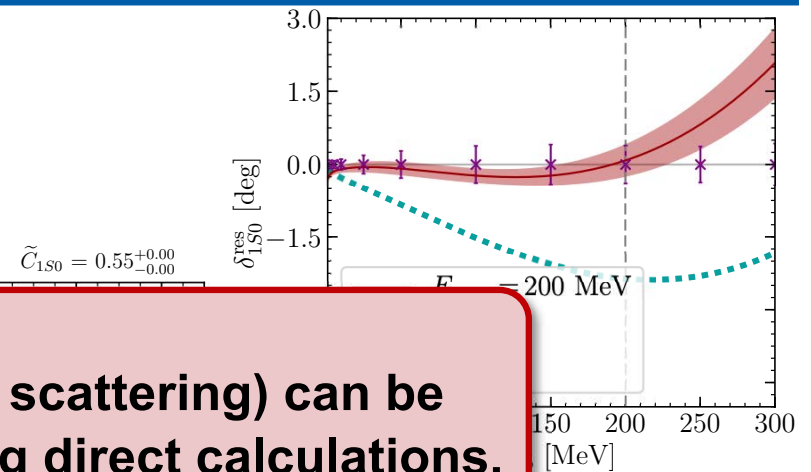


Eigenvector Continuation for scattering with local chiral NN and optical potentials

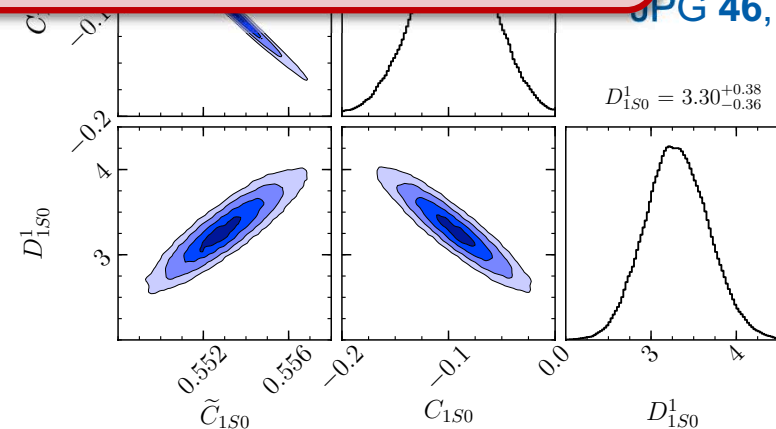
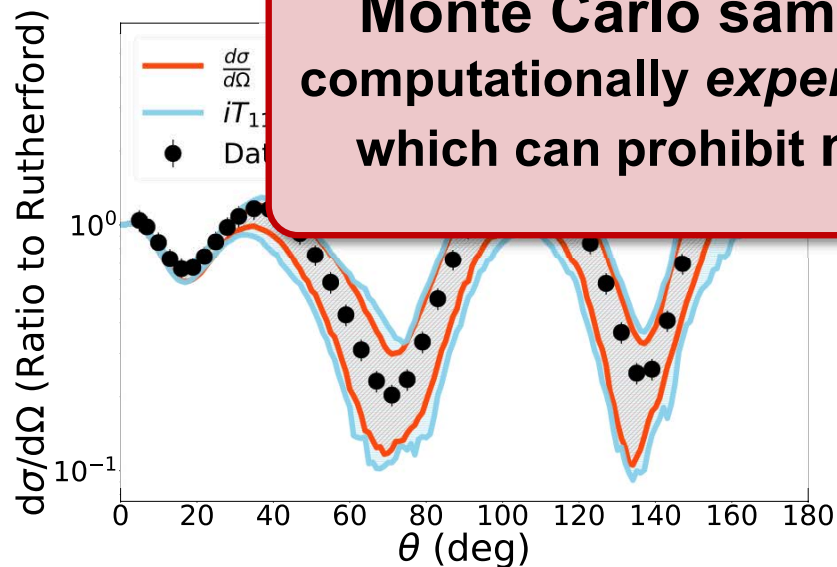
Bayesian uncertainty quantification



Chiral potentials



Monte Carlo sampling (for scattering) can be computationally expensive using direct calculations, which can prohibit meaningful Bayesian UQ.

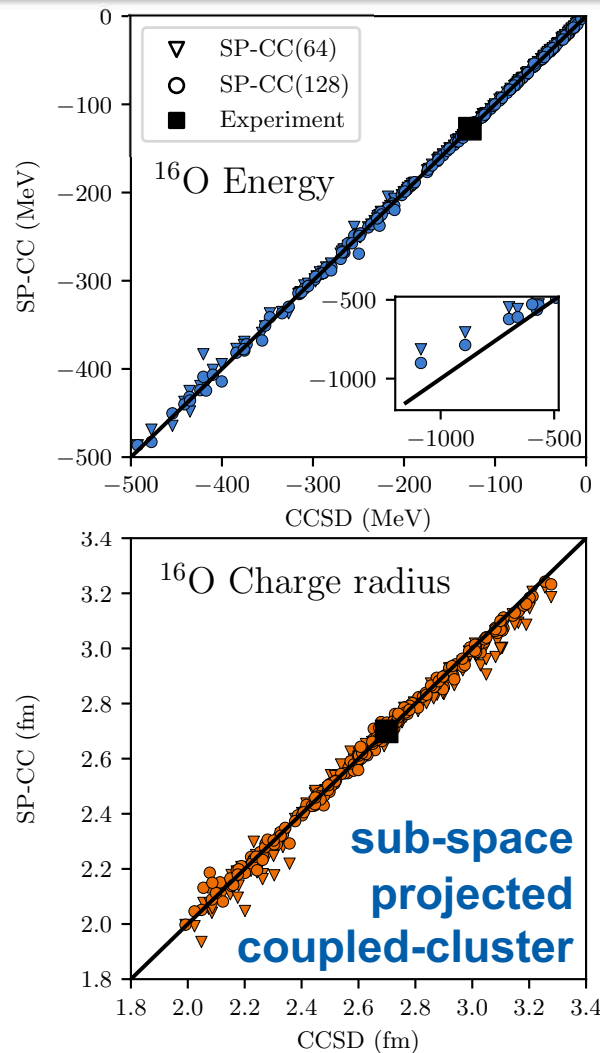


solowski et al.,
PJP 46, 045102

Optical potentials

Eigenvector Continuation for scattering with local chiral NN and optical potentials

Rigorous UQ and global sensitivity analysis

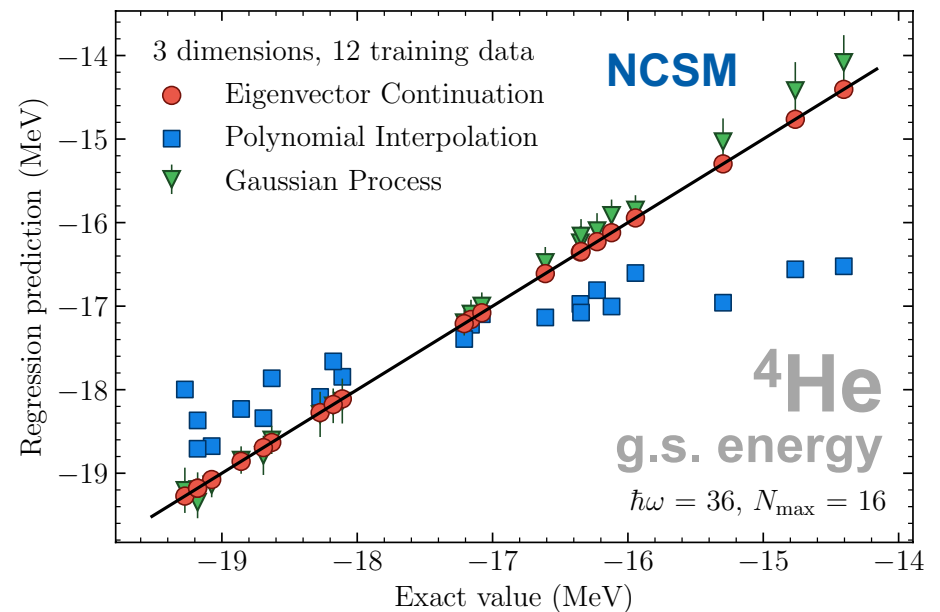


Ekström & Hagen, PRL 123, 252501

Eigenvector Continuation

Frame *et al.*, PRL 121, 032501

accurate & fast emulators
efficient resummation tool



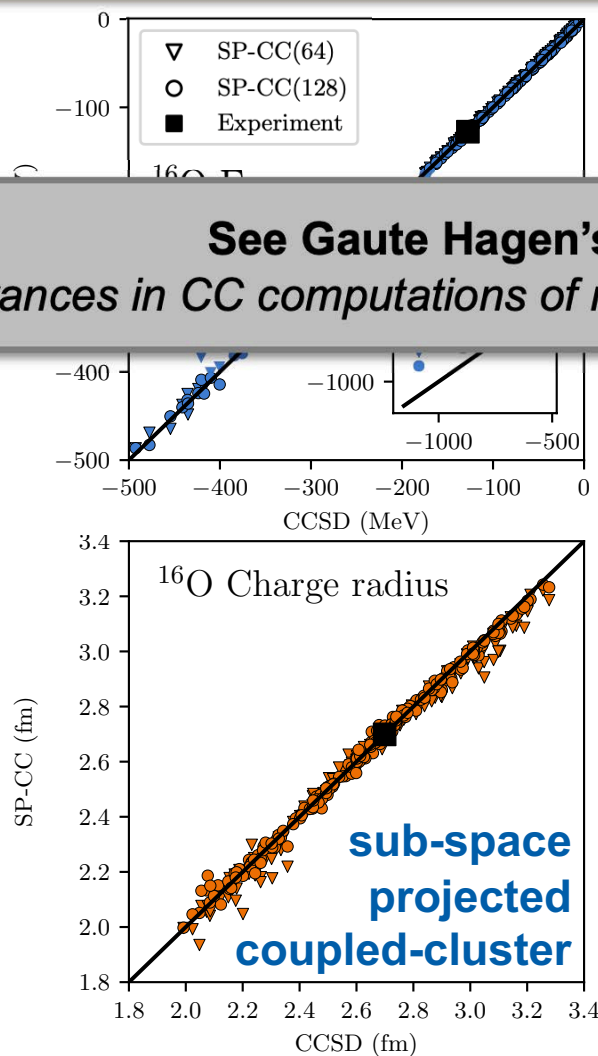
König, Ekström *et al.*, PLB 810, 135814

Demol, Duguet *et al.*, PRC 101, 041302

Wesolowski, Svensson *et al.*, arXiv:2104.04441

Eigenvector Continuation for scattering with local chiral NN and optical potentials

Rigorous UQ and global sensitivity analysis

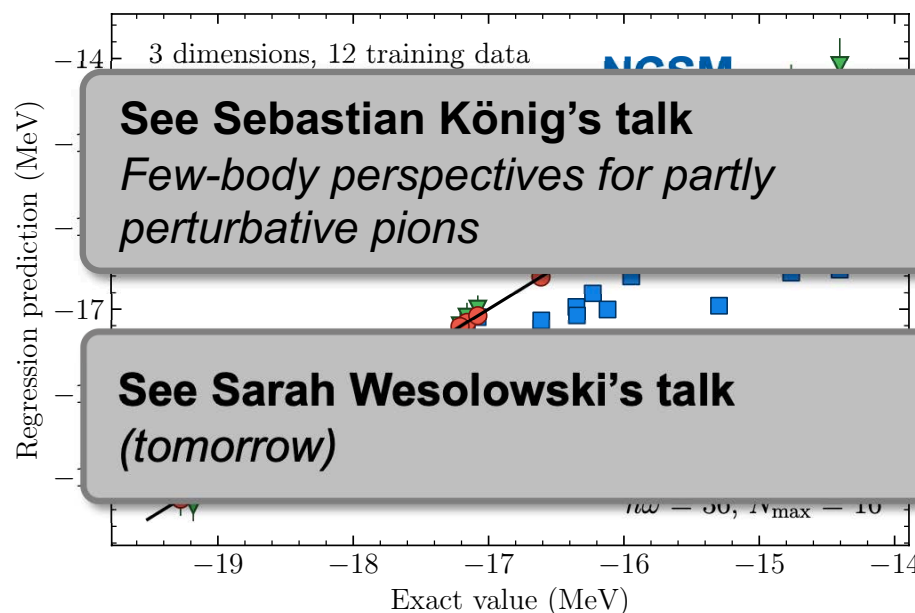


See Gaute Hagen's talk
Advances in CC computations of nuclei

Eigenvector Continuation

Frame *et al.*, PRL 121, 032501

accurate & fast emulators
efficient resummation tool



See Sebastian König's talk
Few-body perspectives for partly perturbative pions

See Sarah Wesolowski's talk
(tomorrow)

König, Ekström *et al.*, PLB 810, 135814

Demol, Duguet *et al.*, PRC 101, 041302

Wesolowski, Svensson *et al.*, arXiv:2104.04441

Ekström & Hagen, PRL 123, 252501

Eigenvector Continuation for scattering with local chiral NN and optical potentials

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Kohn Variational Principle

Furnstahl, Garcia, Millican, and Zhang, PLB 809, 10135719

$$\beta [|\psi_{\text{trial}}\rangle] = \frac{K_\ell}{p} - 2\mu \langle \psi_{\text{trial}} | H(\boldsymbol{\theta}) - E | \psi_{\text{trial}} \rangle \quad p = \sqrt{2\mu E}$$

stationary approximation to exact K_ℓ matrix [accurate up to $O(\delta u^2)$]

$$u_{\ell,E}^{\text{trial}}(r) \sim \frac{1}{p} \sin(\eta_\ell) + \frac{K_\ell}{p} \cos(\eta_\ell) \quad \eta_\ell = kr - \frac{\pi}{2}\ell \quad H(\boldsymbol{\theta}) = T + V(\boldsymbol{\theta})$$



for EC in R matrix theory,
see also Bai & Ren, PRC **103**, 014612

Eigenvector Continuation for scattering with local chiral NN and optical potentials

Kohn Variational Principle **1**

Furnstahl, Garcia, Millican, and Zhang, PLB 809, 10135719

2

$$\beta[|\psi_{\text{trial}}\rangle] = \frac{K_\ell}{p} - 2\mu \langle \psi_{\text{trial}} | H(\boldsymbol{\theta}) - E | \psi_{\text{trial}} \rangle \quad p = \sqrt{2\mu E}$$

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3

4

Training: solve RSE *exactly* for a set $\{\boldsymbol{\theta}_i\}_{i=1}^{N_b}$ and construct the trial wave function:

$$|\psi_{\text{trial}}\rangle = \sum_{i=1}^{N_b} c_i |\psi_E(\boldsymbol{\theta}_i)\rangle$$

Given $H(\boldsymbol{\theta})$, the stationary point is obtained by simple linear algebra:

$$\delta\beta[|\psi_{\text{trial}}\rangle] = 0 \quad \text{s.t.} \quad \sum_{i=1}^{N_b} c_i = 1$$

$$c_i = \sum_j (\Delta\tilde{U})_{ij}^{-1} \left(\frac{K_\ell^{(j)}(E)}{p} - \lambda \right) \quad \lambda = \frac{-1 + \sum_{ij} (\Delta\tilde{U})_{ij}^{-1} \frac{K_\ell^{(j)}(E)}{p}}{\sum_{ij} (\Delta\tilde{U})_{ij}^{-1}}$$

with matrix $\Delta\tilde{U}_{ij} = 2\mu \langle \psi_E(\boldsymbol{\theta}_i) | 2V(\boldsymbol{\theta}) - V(\boldsymbol{\theta}_i) - V(\boldsymbol{\theta}_j) | \psi_E(\boldsymbol{\theta}_j) \rangle$ **5**

Approximate $K_\ell = \tan \delta_\ell$: $[K_\ell(E)]_{\text{exact}} \approx \sum_i c_i K_\ell^{(i)}(E) - \frac{p}{2} \sum_{ij} c_i \Delta\tilde{U}_{ij} c_j$

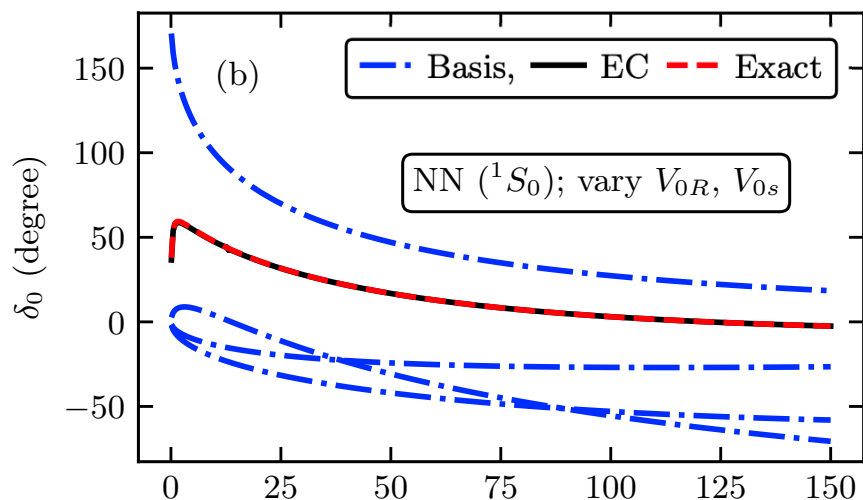
emulate

validate



Eigenvector Continuation for scattering with local chiral NN and optical potentials

Furnstahl, Garcia, Millican, and Zhang, PLB 809, 10135719

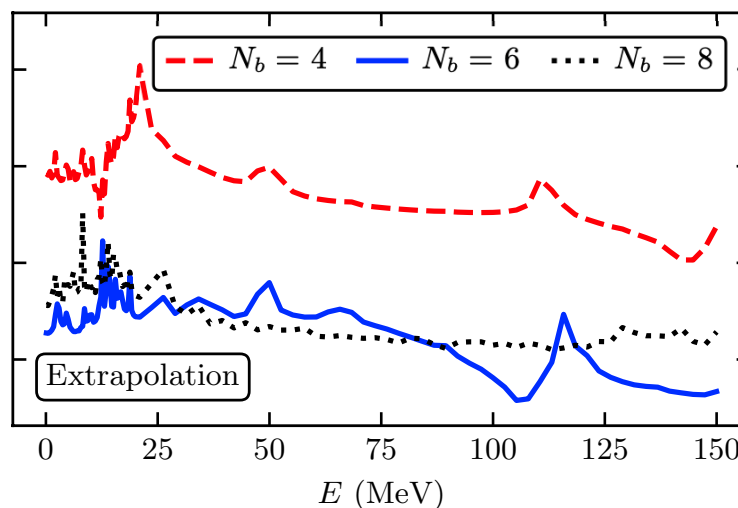
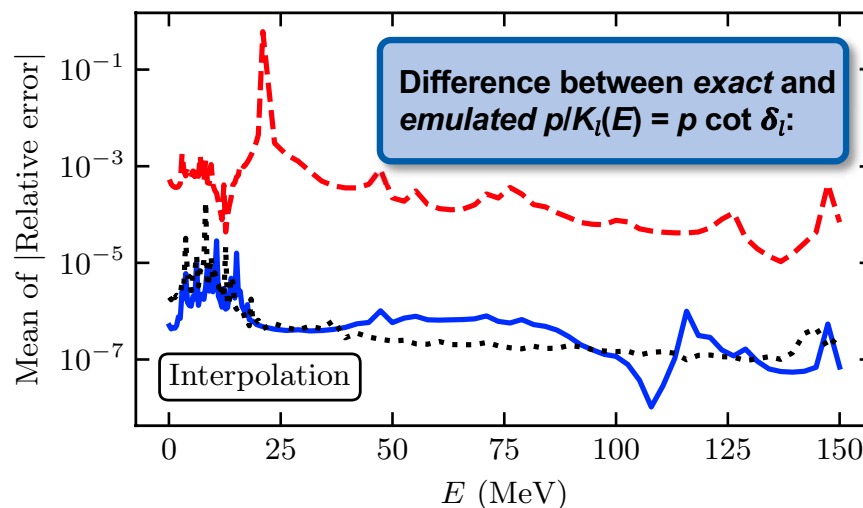


EC for scattering is effective for inter- and extrapolation

- local & nonlocal potentials
- incl. optical potentials
- high partial waves

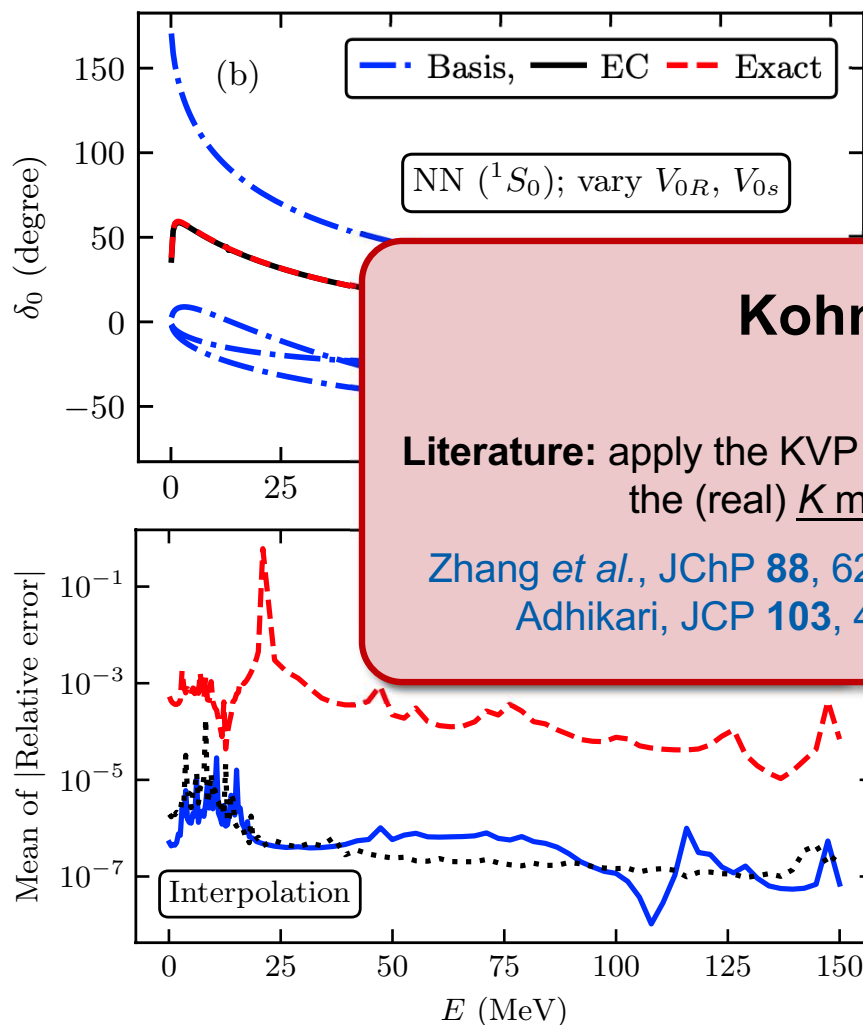
Minnesota interaction:

$$V_{1S_0}(r) \equiv V_{0R}e^{-\kappa_R r^2} + V_{0s}e^{-\kappa_s r^2}$$



Eigenvector Continuation for scattering with local chiral NN and optical potentials

Furnstahl, Garcia, Millican, and Zhang, PLB 809, 10135719



EC for scattering is effective for inter- and extrapolation

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Kohn anomalies

Schwartz, PR 124, 1468 (1961)

Literature: apply the KVP to the (complex) S matrix rather than the (real) K matrix to avoid anomalies

Zhang *et al.*, JChP 88, 6233

Adhikari, JCP 103, 415

Aymar *et al.*, RMP 68, 1015

Nesbet (Plenum Press, 1980)



Eigenvector Continuation for scattering with local chiral NN and optical potentials

Generalized Kohn Variational Principle

Lucchese, PRA 40, 112

$$\beta_u [|\psi_{\text{trial}}\rangle] = L_\ell - \frac{2\mu}{\det u} \langle \psi_{\text{trial}} | H(\boldsymbol{\theta}) - E | \psi_{\text{trial}} \rangle$$

stationary approximation to exact L_ℓ matrix [accurate up to $O(\delta u^2)$]

$$u_{\ell,E}^{\text{trial}}(r) \sim \phi_0(r) + L \phi_1(r) \quad \begin{aligned} \phi_0(r) &\sim u_{00} \sin(\eta_\ell) + u_{01} \cos(\eta_\ell) \\ \phi_1(r) &\sim u_{10} \sin(\eta_\ell) + u_{11} \cos(\eta_\ell) \end{aligned} \quad \eta_\ell = kr - \frac{\pi}{2}\ell$$

$$\begin{aligned} L = K &\text{ for } u = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ L = S &\text{ for } u = \begin{bmatrix} i & -1 \\ i & 1 \end{bmatrix} \\ L = T &\text{ for } u = \begin{bmatrix} 1 & 0 \\ i & 1 \end{bmatrix} \end{aligned}$$

... any nonsingular matrix

$$K = \frac{u_{01} + u_{11}L}{u_{00} + u_{10}L}$$



for applications to N-d and p-d scattering, see
 Viviani, Kievsky, and Rosati, FBS 30, 3
 Kievsky, NPA 624, 125
 Kievsky, Viviani, and Rosati, NPA 577, 511

Eigenvector Continuation for scattering with local chiral NN and optical potentials

Generalized Kohn Variational Principle

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Given $H(\boldsymbol{\theta})$, the stationary point is obtained by simple linear algebra: $\delta\beta[\psi_{\text{trial}}] = 0$ s.t. $\sum_{i=1}^{N_b} c_i = 1$

$$c_i = \sum_j (\Delta\tilde{U})_{ij}^{-1} \left(L_\ell^{(j)}(E) - \lambda \right) \quad \lambda = \frac{-1 + \sum_{ij} (\Delta\tilde{U})_{ij}^{-1} L_\ell^{(j)}(E)}{\sum_{ij} (\Delta\tilde{U})_{ij}^{-1}}$$

with matrix $\Delta\tilde{U}_{ij} = \frac{2\mu}{\det u} \langle \psi_E(\boldsymbol{\theta}_i) | 2V(\boldsymbol{\theta}) - V(\boldsymbol{\theta}_i) - V(\boldsymbol{\theta}_j) | \psi_E(\boldsymbol{\theta}_j) \rangle$

Approximate L_ℓ : $[L_\ell(E)]_{\text{exact}} \approx \sum_i c_i L_\ell^{(i)}(E) - \frac{1}{2} \sum_{ij} c_i \Delta\tilde{U}_{ij} c_j$

emulate

validate



Eigenvector Continuation for scattering with local chiral NN and optical potentials

Generalized KVP in practice

CD, Giuliani, Quinonez *et al.*, in prep.

$$[L_\ell(E)]_{\text{exact}} \approx \sum_i c_i L_\ell^{(i)}(E) - \frac{1}{2} \sum_{ij} c_i \Delta \tilde{U}_{ij} c_j$$

Considered a wide range of KVPs:

- S , T , K , their inverses, and random complex parametrizations u
- uncoupled channels
- local coordinate-space potentials

Potentials $V(r; \theta)$ implemented:

- Minnesota (real, nonlinear in θ)
- Woods-Saxon (real, nonlinear in θ)
- Optical (complex, nonlinear in θ)
- **Local chiral GT+ (real, linear in θ)**

We sample $H(\theta)$ randomly for different channels, energies, and basis sizes, and **investigate deviations of emulated from exact phase shifts.**

To measure the efficacy of EC, we fit the c_i 's to the exact wave function.

$$|\psi_{\text{trial}}\rangle = \sum_{i=1}^{N_b} c_i |\psi_E(\theta_i)\rangle$$

The EC basis usually is still effective in energy regions where the KVP is affected by Kohn anomalies.

Eigenvector Continuation for scattering with local chiral NN and optical potentials

Complex vs real KVP

CD, Giuliani, Quinonez *et al.*, in prep.

Why is the complex KVP less prone to Kohn anomalies?

$$\lambda = \frac{-1 + \sum_{ij} (\Delta \tilde{U})_{ij}^{-1} L_\ell^{(j)}(E)}{\sum_{ij} (\Delta \tilde{U})_{ij}^{-1}}$$

There are several sources of Kohn anomalies. For instance, no stationary point of the functional can be found if

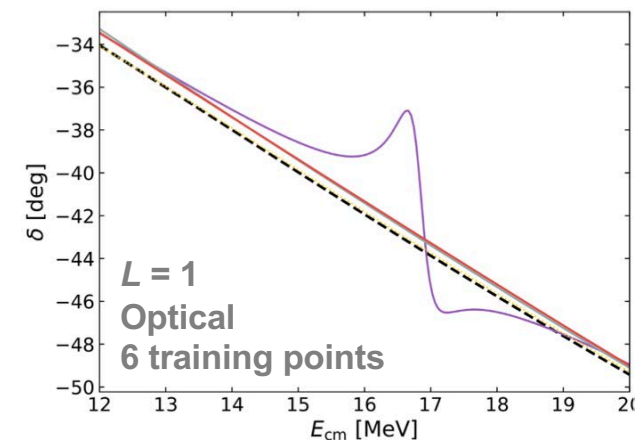
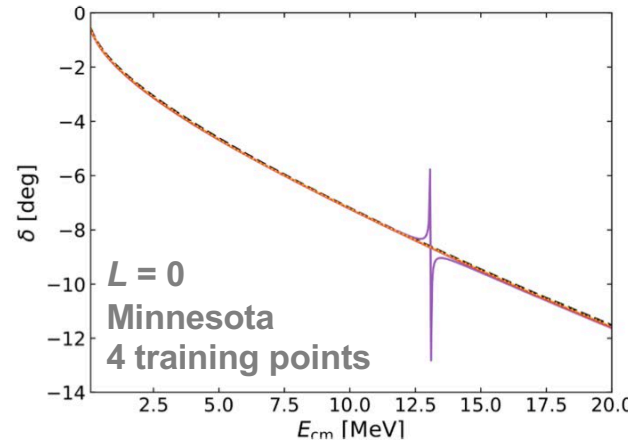
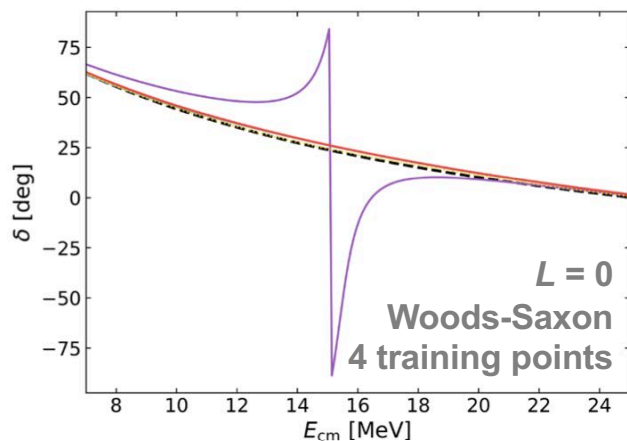
$$\sum_{ij} (\Delta \tilde{U})_{ij}^{-1} = 0$$

(can be used to pinpoint anomalies)

Advantage: the denominator is complex for the complex KVP and/or optical potentials, hence improving issues with zero crossings

No guarantee: we found Kohn anomalies in the *K* matrix for all but **chiral potentials**.

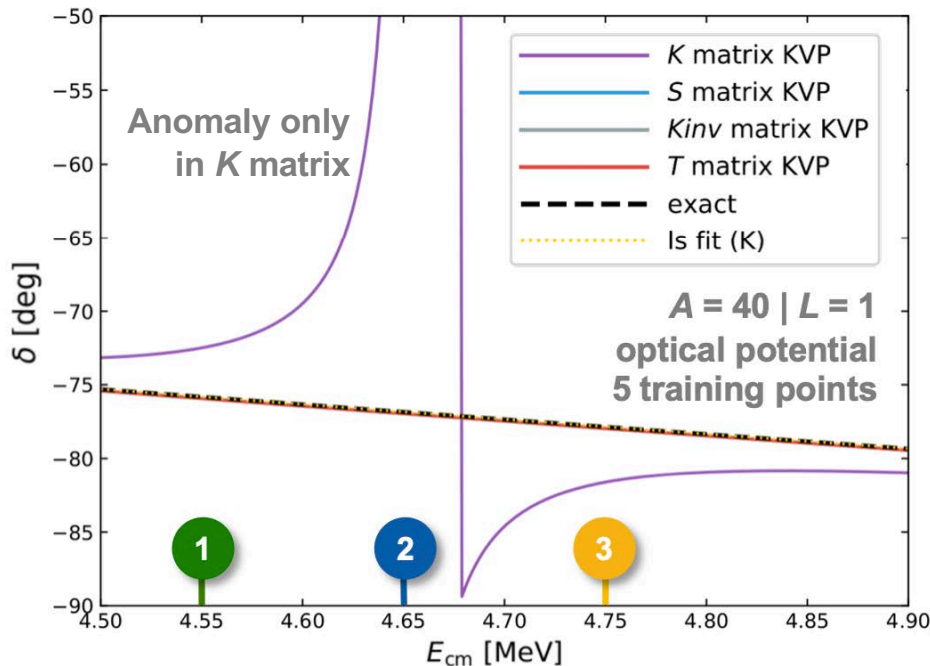
Other sources: bound states, resonances...



Eigenvector Continuation for scattering with local chiral NN and optical potentials

Phase shifts and (differential) cross sections

CD, Giuliani, Quinonez *et al.*, in prep.

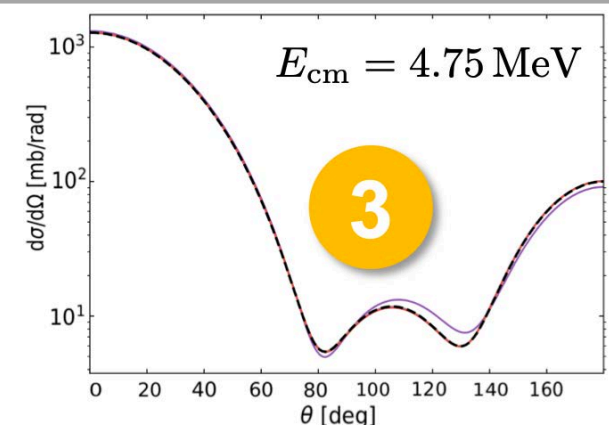
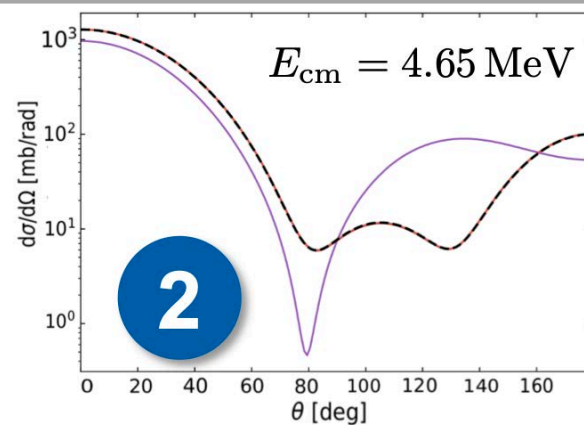
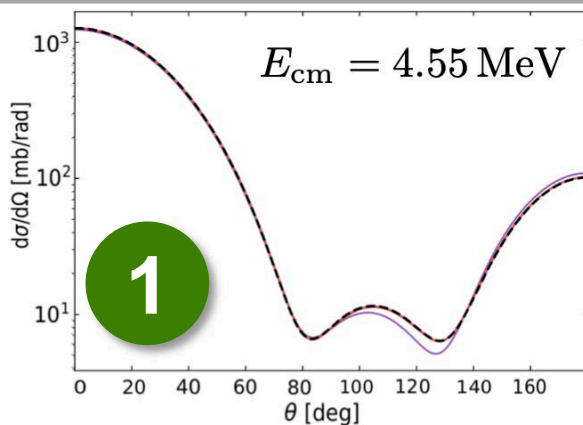


$$\frac{d\sigma}{d\Omega} = \left| \frac{1}{2ik} \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) (S_l - 1) \right|^2$$

Overall, the complex KVP reproduces well phase shifts and cross sections in contrast to the real KVP.

In absence of anomalies, the real KVP may be slightly more accurate for some E

differential cross sections



Eigenvector Continuation for scattering with local chiral NN and optical potentials

Diagnostic tools

see also: Lucchese, PRA 40, 112

Does that imply that the complex KVP is free of *Kohn anomalies*?

$$[L_\ell(E)]_{\text{exact}} \approx \sum_i c_i L_\ell^{(i)}(E) - \frac{1}{2} \sum_{ij} c_i \Delta \tilde{U}_{ij} c_j$$

Diagnostic tools for anomalies:

- Use **multiple KVPs in parallel**
- Check KVPs for consistency

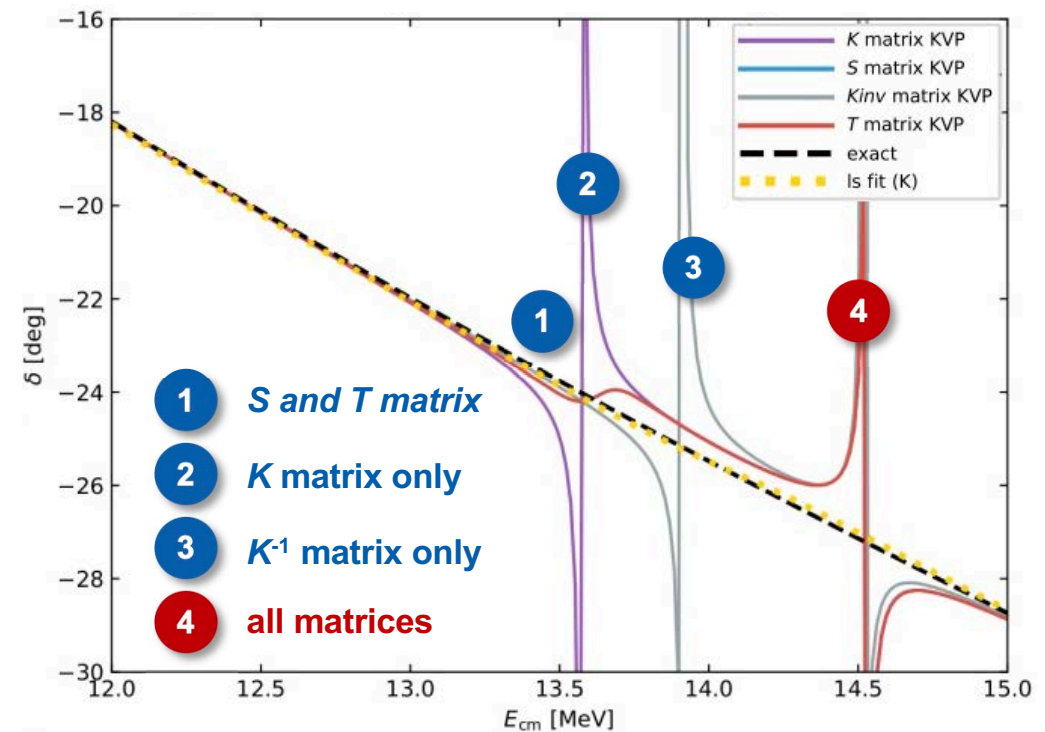
$$S = 1 + 2iT \quad L^{-1}L = 1$$

$$|S| = 1 \quad (\text{for real } V) \quad (\dots)$$

- Track and find zero crossings of

$$\Gamma(E) = \sum_{ij} (\Delta \tilde{U})_{ij}^{-1}$$

- **Change the size of the training basis** (double, add/remove, etc.)



see also: Lucchese, PRA 40, 112
Viviani, Kievsky, and Rosati, FBS 30, 3

Eigenvector Continuation for scattering with local chiral NN and optical potentials

Numerical noise

CD, Giuliani, Quinonez *et al.*, in prep.

The condition number increases with increasing number of training points.

Methods to control the noise in matrix inversions are *not* efficient (fine-tuned)

- nugget regularization
- Moore-Penrose inverse

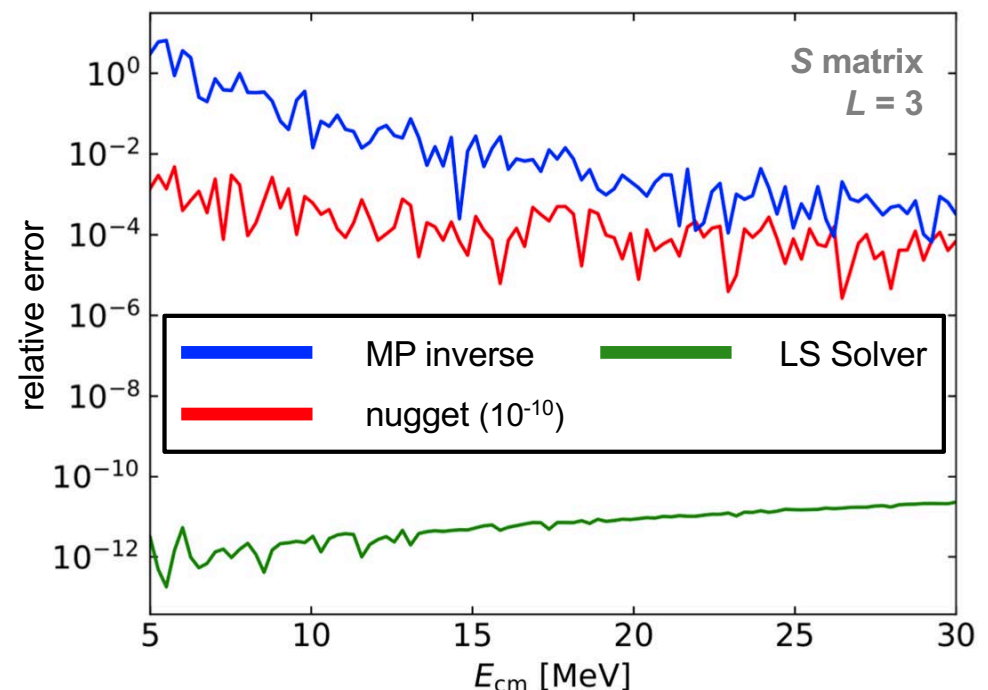
S matrix especially prone to noise, whereas the T matrix is more stable

Simple yet robust: find *stationary point* numerically (e.g., using an LS solver)

$$\begin{pmatrix} \Delta\tilde{U} & 1 \\ \mathbf{1}^T & 0 \end{pmatrix} \begin{pmatrix} \mathbf{c} \\ \lambda \end{pmatrix} = \begin{pmatrix} \mathbf{L} \\ 1 \end{pmatrix}$$

inverse-free,
if possible

$$c_i = \sum_j (\Delta\tilde{U})_{ij}^{-1} \left(L_\ell^{(j)}(E) - \lambda \right)$$
$$\lambda = \frac{-1 + \sum_{ij} (\Delta\tilde{U})_{ij}^{-1} L_\ell^{(j)}(E)}{\sum_{ij} (\Delta\tilde{U})_{ij}^{-1}}$$

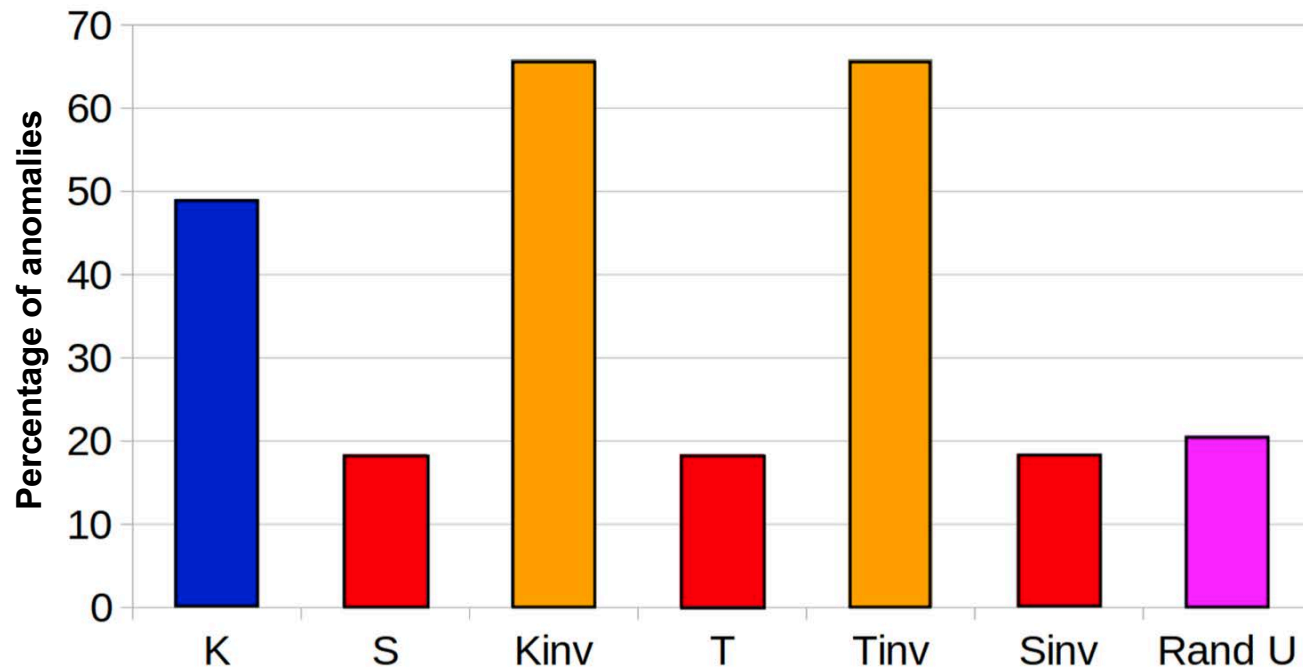


Eigenvector Continuation for scattering with local chiral NN and optical potentials

Abundance of Kohn anomalies

Which KVP performs best in terms of reducing *Kohn anomalies*?

But be careful: this depends on the interaction, channel, ...



15 random matrices with a wide range in $|\det(u)|$

$L = 0$ | $A = 40$ | $E = 5-20$ MeV
Woods-Saxon potential
5 training points

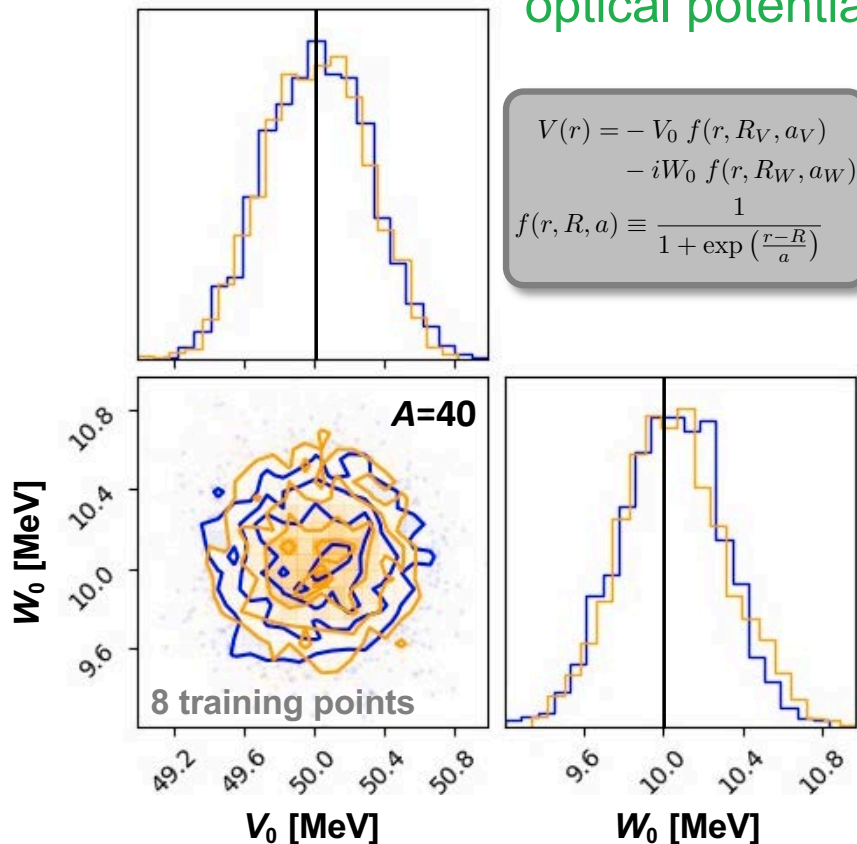
sample size: 500

Eigenvector Continuation for scattering with local chiral NN and optical potentials

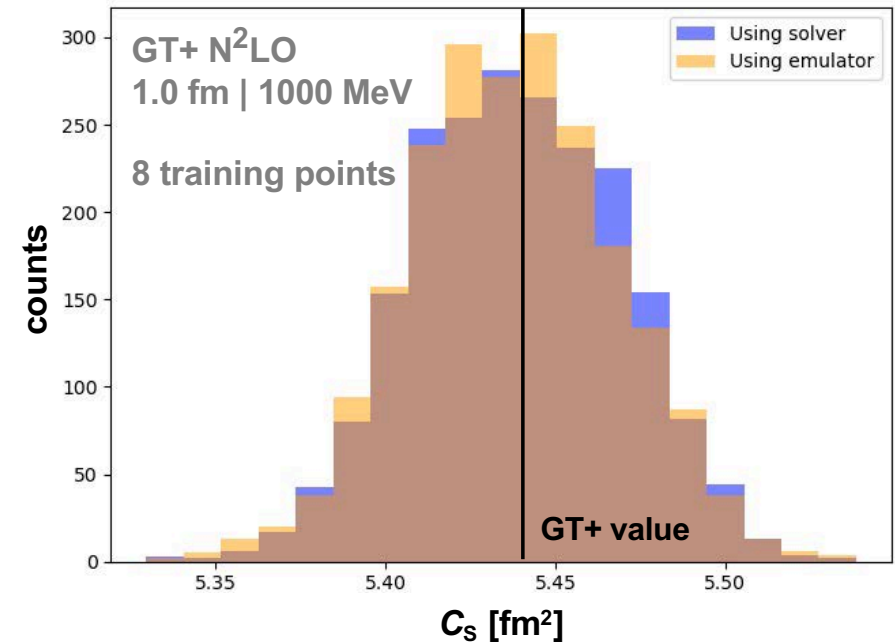
Proof of principle: MCMC sampling

CD, Giuliani, Quinonez *et al.*, in prep.

Possible application: Bayesian model comparison for optical potentials



Local chiral potential



Gezerlis, Tews *et al.*, PRC **90**, 054323

Possible application: parameter estimation for constructing next-generation (local) chiral interactions

Buchner (“UltraNest”), arXiv:2101.09604
needs coupled channels; Kamimura, PTPS **62**, 236

Optical potential

Eigenvector Continuation for scattering with local chiral NN and optical potentials

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Christian Drischler

April 26, 2021 | INT 21-1b: Nuclear Forces for Precision Nuclear Physics

This week's topic:

Improving nuclear forces with *novel fitting strategies* and higher orders in chiral EFT

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using EC as an efficient emulator for scattering with local chiral interactions and optical potentials

2

statistical quantification and propagation of EFT truncation errors in nuclear matter calculations

Keywords:

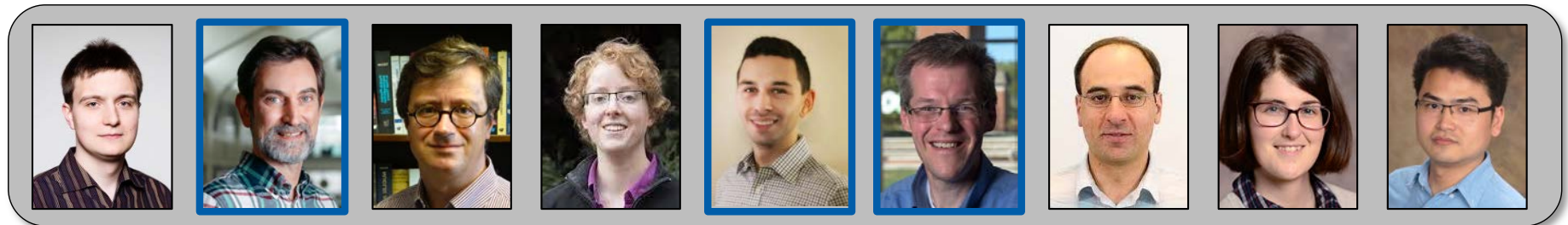
- + ChEFT + scattering
- + Variational principles
- + Eigenvector Continuation
- + Bayesian UQ
- + infinite nuclear matter
- + $N^3\text{LO}$ NN + 3N forces
- + ...

Eigenvector Continuation for scattering with local chiral NN and optical potentials

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New framework for UQ of the infinite-matter EOS

buqeye.github.io

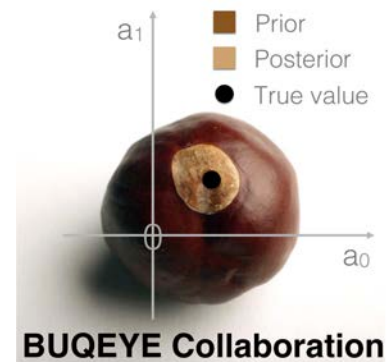


CD, Furnstahl, Melendez, and Phillips

*How well do we know the neutron-matter equation of state at the densities inside neutron stars? A Bayesian approach with correlated uncertainties, PRL **125**, 202702*

CD, Melendez, Furnstahl, and Phillips

*Effective Field Theory Convergence Pattern of Infinite Nuclear Matter, PRC **102**, 054315*



**Bayesian
Uncertainty
Quantification:
Errors for
Your
EFT**

**UQ framework available at
<https://buqeye.github.io>**

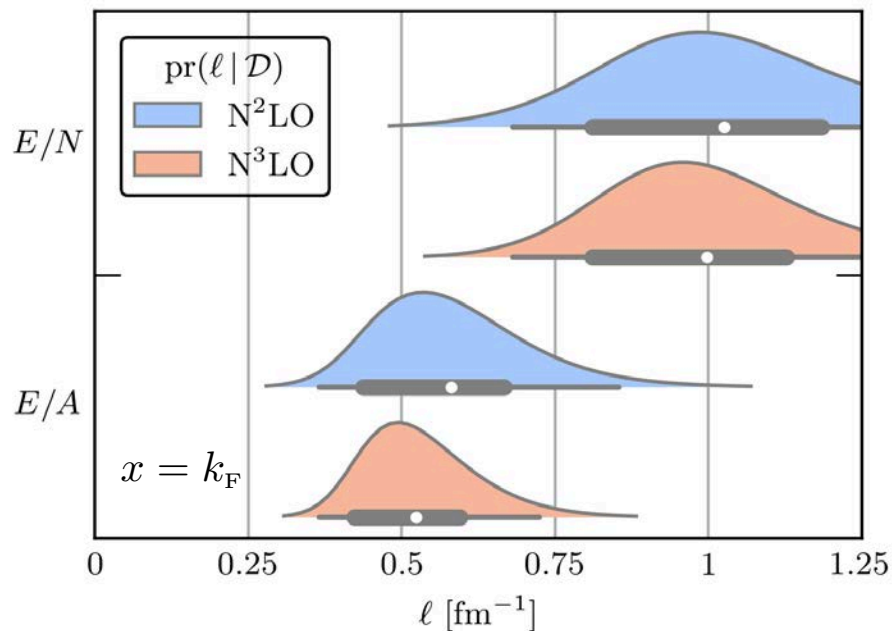
Eigenvector Continuation for scattering with local chiral NN and optical potentials

Propagating type-x uncertainties

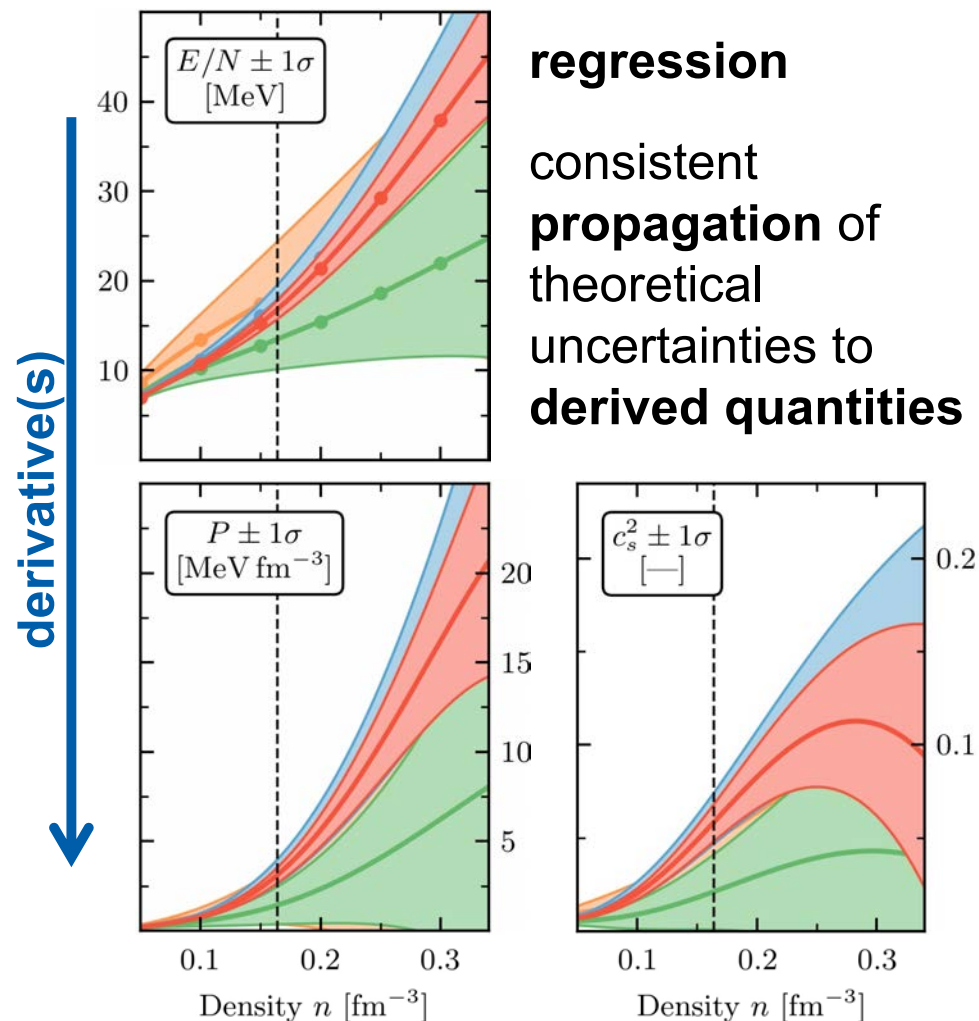
CD, Melendez *et al.*, PRC 102, 054315

How correlated is nuclear matter ?

$\text{pr}(\ell | \mathcal{D})$
correlation length



to be compared with $k_F^{\text{max}} = \begin{cases} 2.2 \text{ fm}^{-1} & \text{PNM} \\ 1.7 \text{ fm}^{-1} & \text{SNM} \end{cases}$



Eigenvector Continuation for scattering with local chiral NN and optical potentials

Bayesian inference

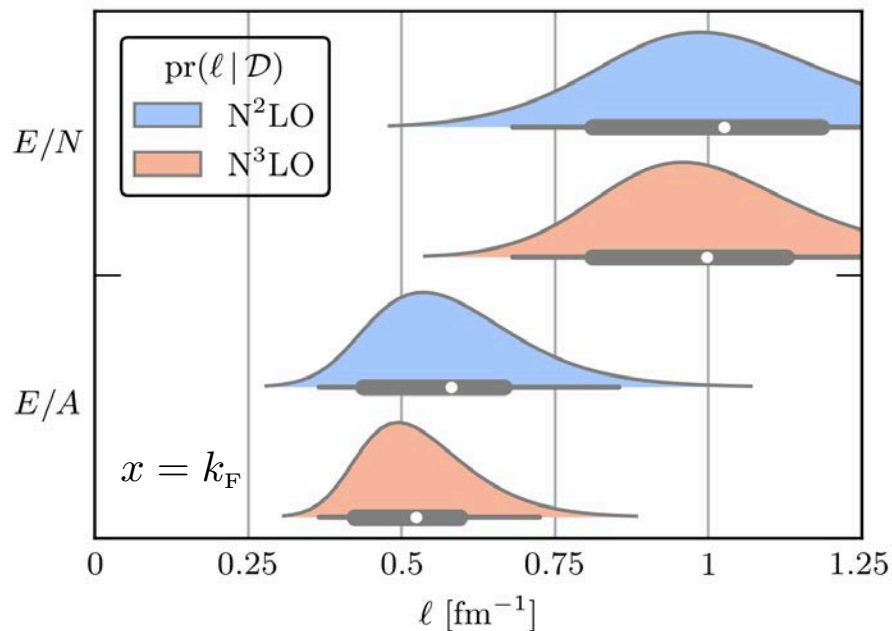
CD, Melendez *et al.*, PRC 102, 054315

How correlated is nuclear matter ?

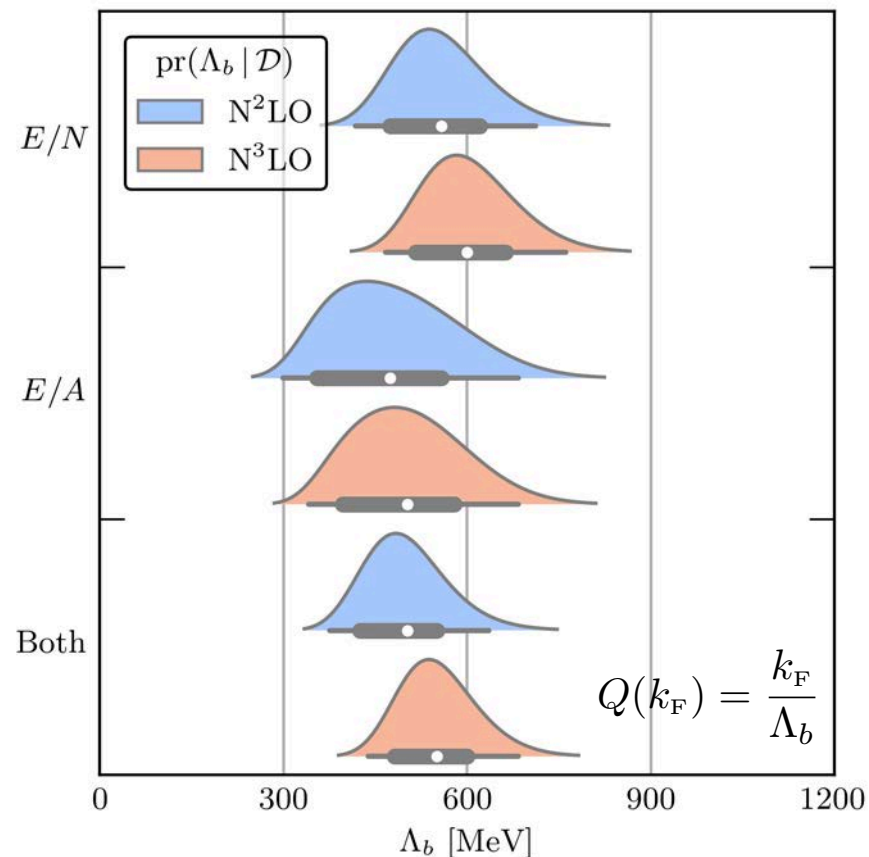
$\text{pr}(\ell | \mathcal{D})$
correlation length

Where does the EFT break down ?

$\text{pr}(\Lambda_b | \mathcal{D})$
breakdown scale



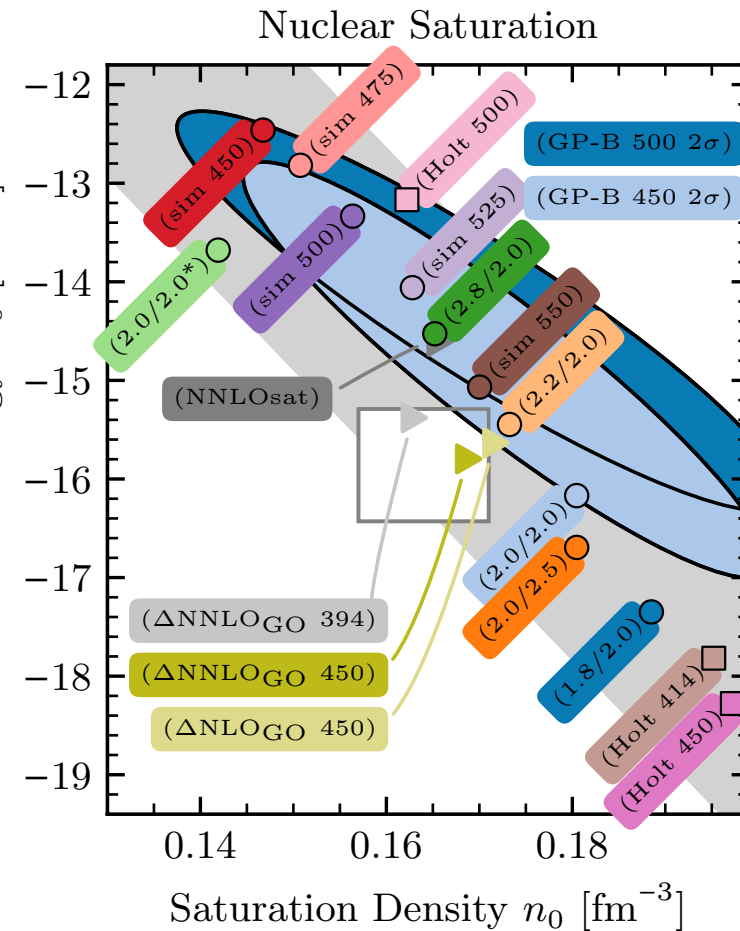
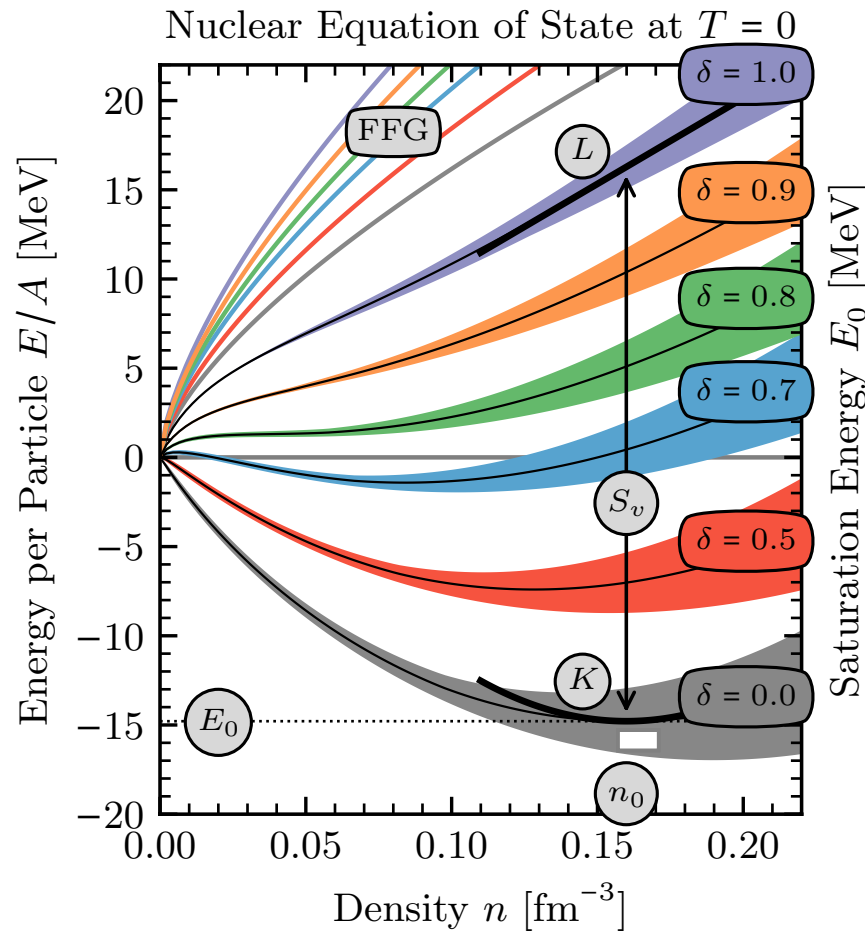
to be compared with $k_F^{\text{max}} = \begin{cases} 2.2 \text{ fm}^{-1} & \text{PNM} \\ 1.7 \text{ fm}^{-1} & \text{SNM} \end{cases}$



Eigenvector Continuation for scattering with local chiral NN and optical potentials

Parameters of the low-density EOS

CD, Holt, and Wellenhofer, arXiv:2101.01709



FFG: free Fermi gas; $\delta = (n_n - n_p)/n$: isospin asymmetry

Annotations: (λ / Λ_{3N}) in fm^{-1} or (Λ) in MeV

for nuclear saturation, see also Atkinson *et al.*, PRC **102**, 044333; Dewulf *et al.*, PRL **90**, 152501

Eigenvector Continuation for scattering with local chiral NN and optical potentials

Summary

1

Presented an efficient emulator using EC and complex KVP

- with local chiral nucleon-nucleon interactions and optical potentials
- KVPs for the T matrix and S matrix can reproduce well phase shifts and cross sections
- EC is efficient in providing trial wave functions for interpolation and extrapolation

2

Studied diagnostic tools for Kohn anomalies

- Kohn anomalies can be encountered in variational calculations
- Consistency checks and variable basis sizes are key to Bayesian UQ
- An improved method for finding stationary points reduces numerical noise substantially

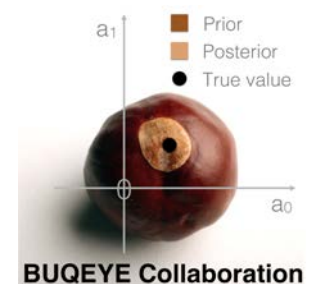
3

Set a new standard for UQ in infinite-matter calculations

- Need for *statistically* robust comparisons between theory, observation, and experiment
- Correlations within *and* between observables are crucial for reliable UQ
- Efficiently quantify and propagate EOS uncertainties to derived quantities

Thanks to my collaborators:

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R. Furnstahl J. Melendez K. McElvain D. Phillips



Eigenvector Continuation for scattering with local chiral NN and optical potentials

Some topics for the discussion session

?

Convergence of Eigenvector Continuation

Avik Sarkar & Dean Lee, PRL **126**, 032501 | Can scattering provide general insights?

?

Extension to three-body scattering with EC

See Xilin Zhang's talk on Wednesday

?

Improved variational principles: insights from pre-EC era?

e.g., Viviani, Kievsky, and Rosati, FBS **30**, 3 and Zhang *et al.*, JChP **88**, 6233

?

Improving nuclear forces with novel fitting strategies

How would the workflow look like in practice?