

Recent progress in the nuclear-matter equation of state from chiral EFT

Christian Drischler

Next Frontiers in the Search for Dark Matter | September 27, 2019

astrophysics
neutron stars,
supernovae, ...

**effective
field
theory**

nuclear physics
structure & reactions
of nuclei

nuclear matter

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ENERGY

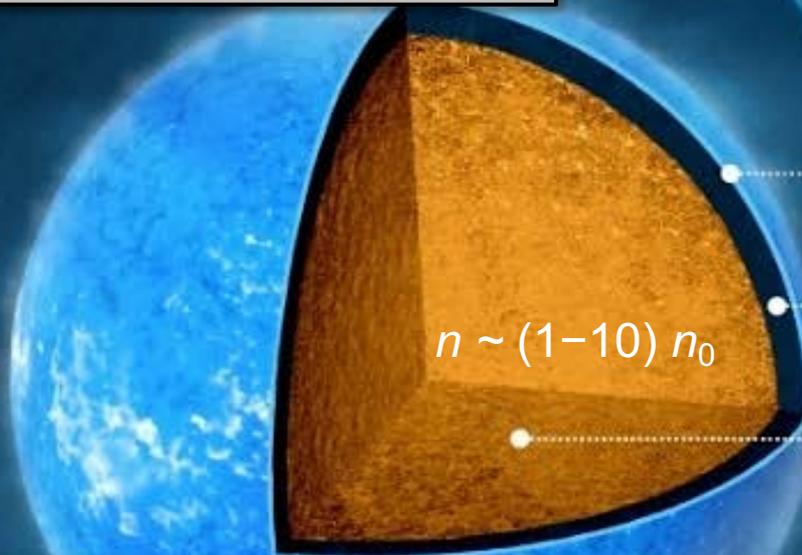


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Neutron stars

e.g., Watts et al., Rev. Mod. Phys. 88, 021001

see also talk by T. Linden



$$n \sim (1-10) n_0$$

$$R \sim (10-14) \text{ km}$$

$$M \sim (1.4-2.0) M_{\text{sun}}$$

$$n_0 \sim 2.7 \cdot 10^{14} \text{ g cm}^{-3}$$

1 | OUTER CRUST

NUCLEI
ELECTRONS

2 | INNER CRUST

NUCLEI
ELECTRONS
SUPERFLUID NEUTRONS

3 | CORE

SUPERFLUID NEUTRONS
SUPERCONDUCTING PROTONS
HYPERONS?
DECONFINED QUARKS?
COLOR SUPERCONDUCTOR?

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(First) Direct detection of gravitational waves



- + Virgo
- + GEO600
- + ...

multi-messenger
astronomy

see also talks by D. Racco
and D. Croon

Binary Neutron-Star Merger

Nobel Prize 2017

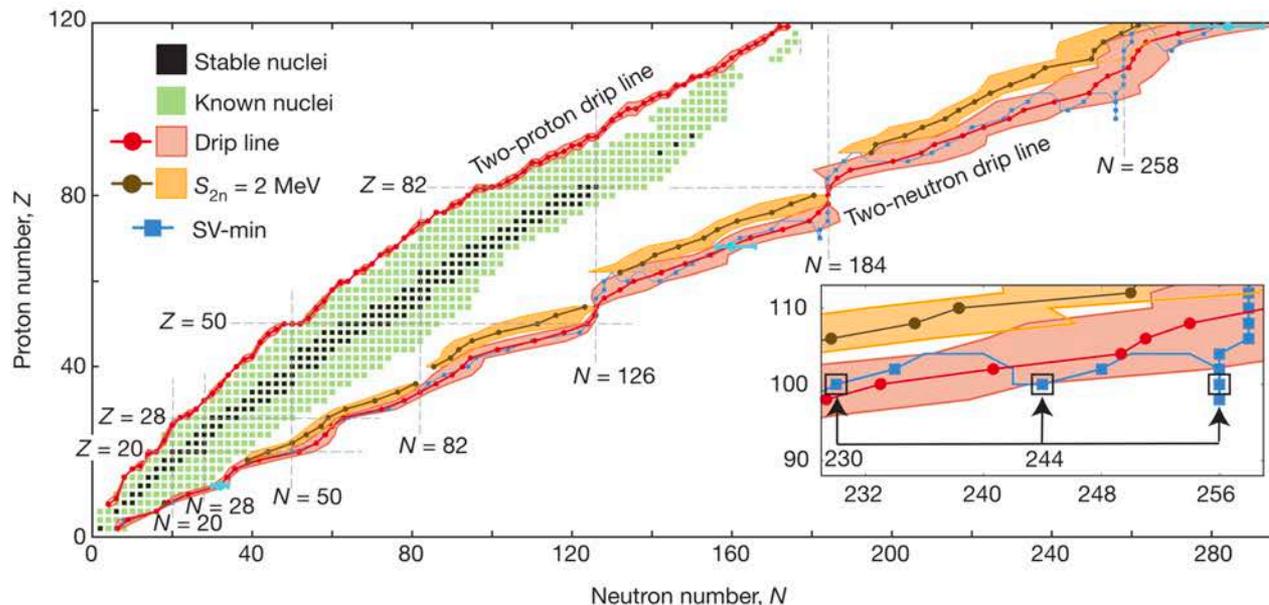


Recent progress in the nuclear-matter equation of state from chiral EFT

Where do heavy elements come from?

How does the nuclear chart emerge from QCD?

How to predict properties of nuclei?



observables

many-body
framework

effective
field theory

quantum
chromodynamics

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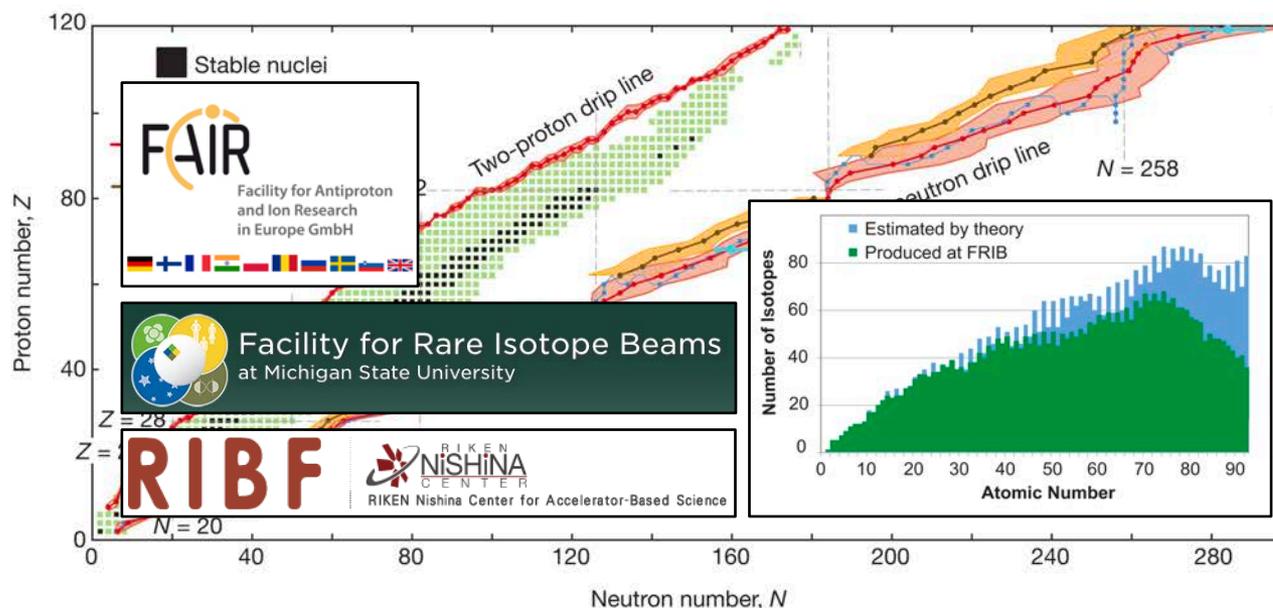
How to predict properties of nuclei?

observables

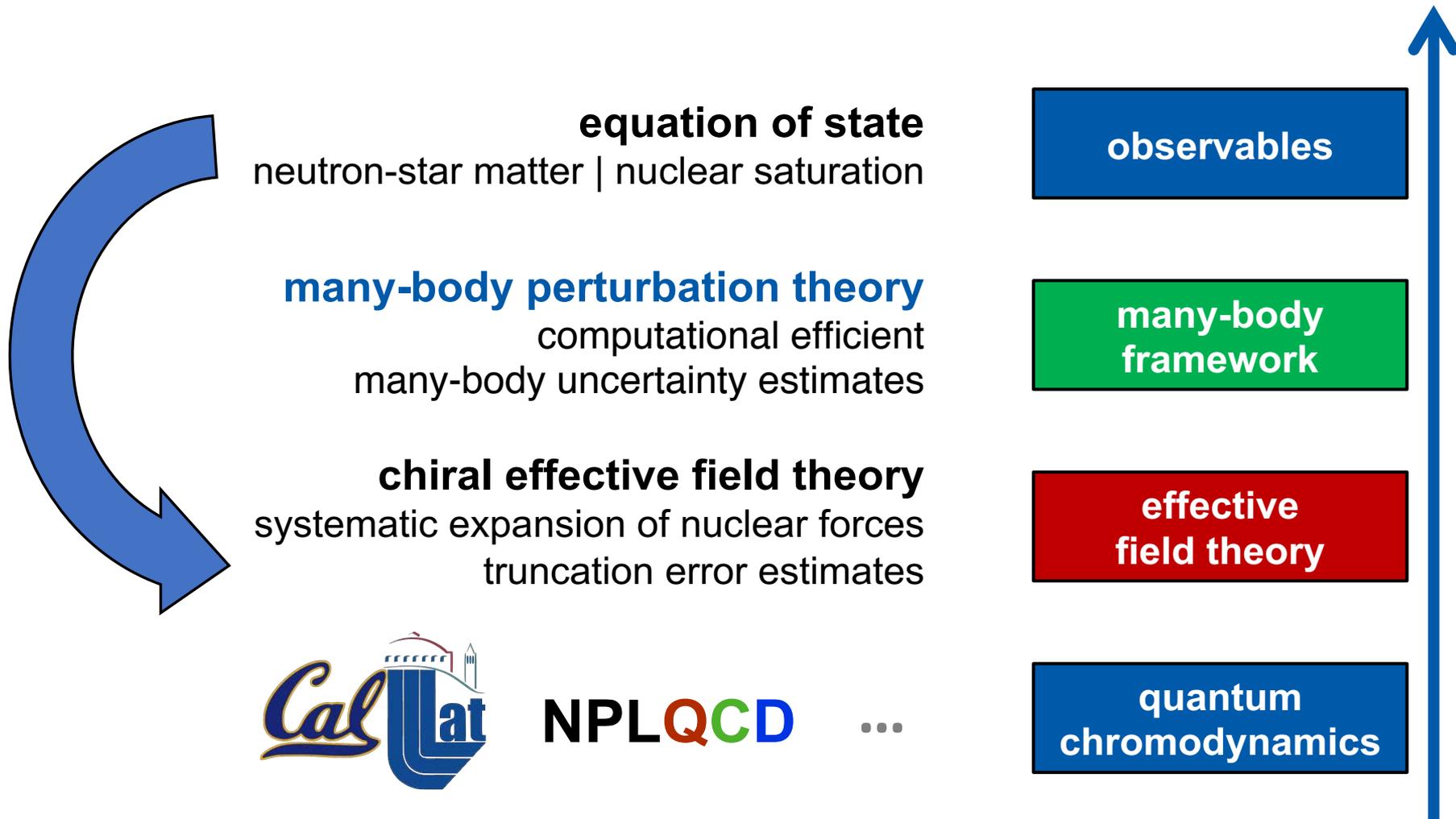
many-body
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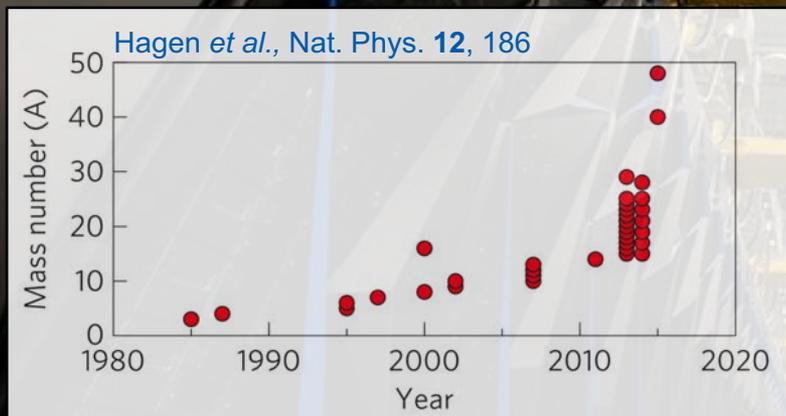
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Recent progress in the nuclear-matter equation of state from chiral EFT

Next-generation supercomputers

#1



202 752 CPU Cores
27 648 NVIDIA GPUs

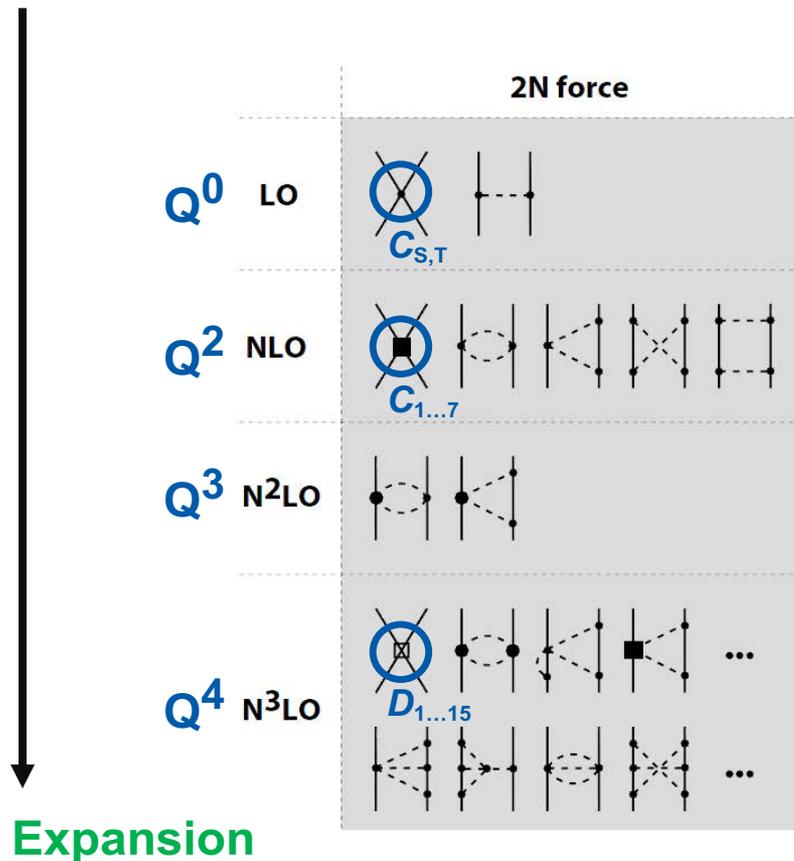
122.3 peta flops

Summit @ ORNL

Recent progress in the nuclear-matter equation of state from chiral EFT

Hierarchy of nuclear forces in chiral EFT

e.g., Machleidt, Entem, Phys. Rep. 503, 1



modern approach to nuclear forces:

- QCD is nonperturbative at the low-energy scales of nuclear physics
- use relevant instead of the fundamental degrees of freedom: e.g., **nucleons** and **pions**
- **pion exchanges** and short-range **contact interactions** (\propto LEC)
- systematic expansion enables improvable **uncertainty estimates**

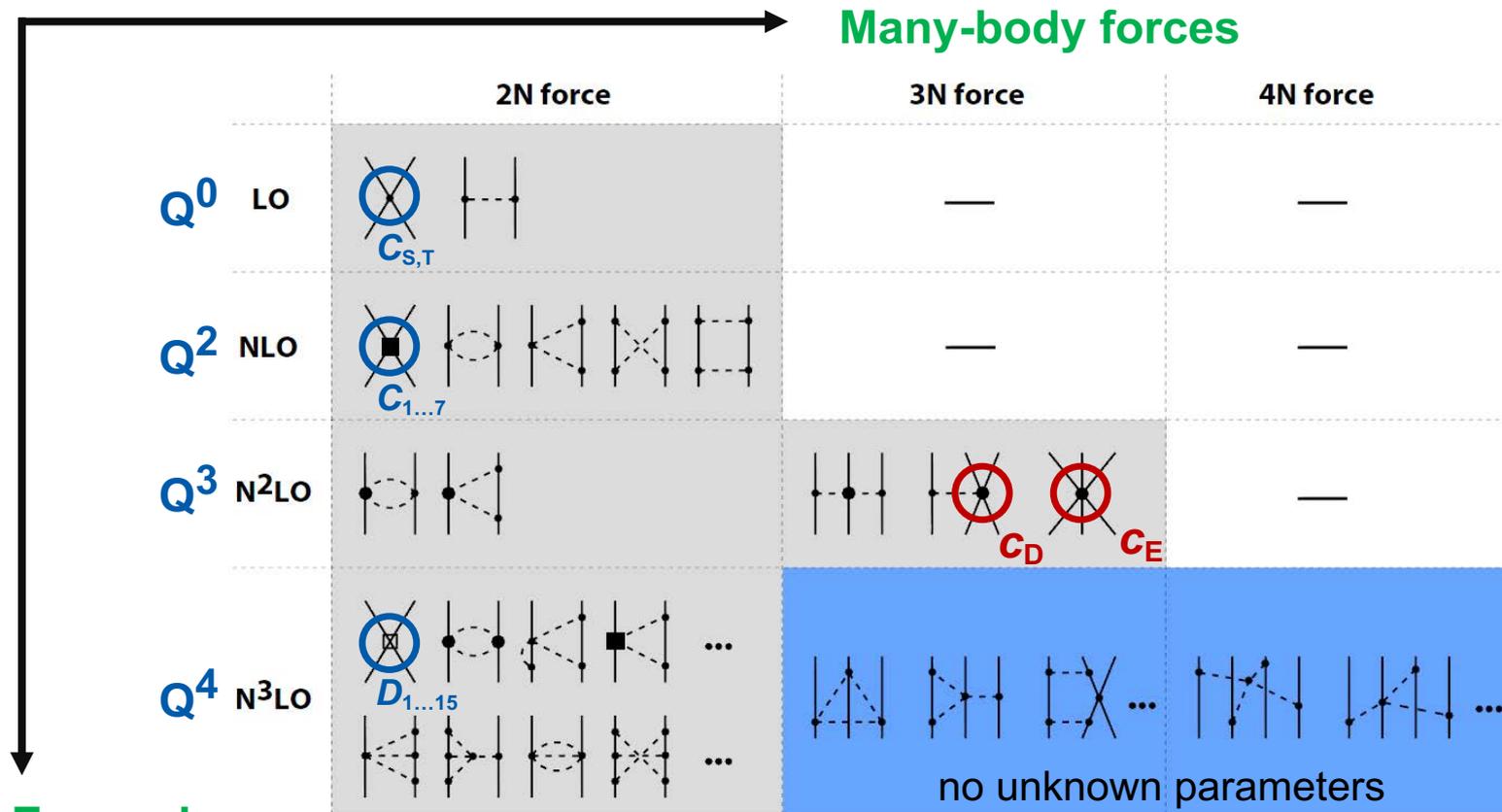
$$Q = \max \left(\frac{p}{\Lambda_b}, \frac{m_\pi}{\Lambda_b} \right) \sim \frac{1}{3}$$

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Krebs, Machleidt, Meißner, ...

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Hierarchy of nuclear forces in chiral EFT

e.g., Machleidt, Entem, Phys. Rep. 503, 1



S. Weinberg

Expansion

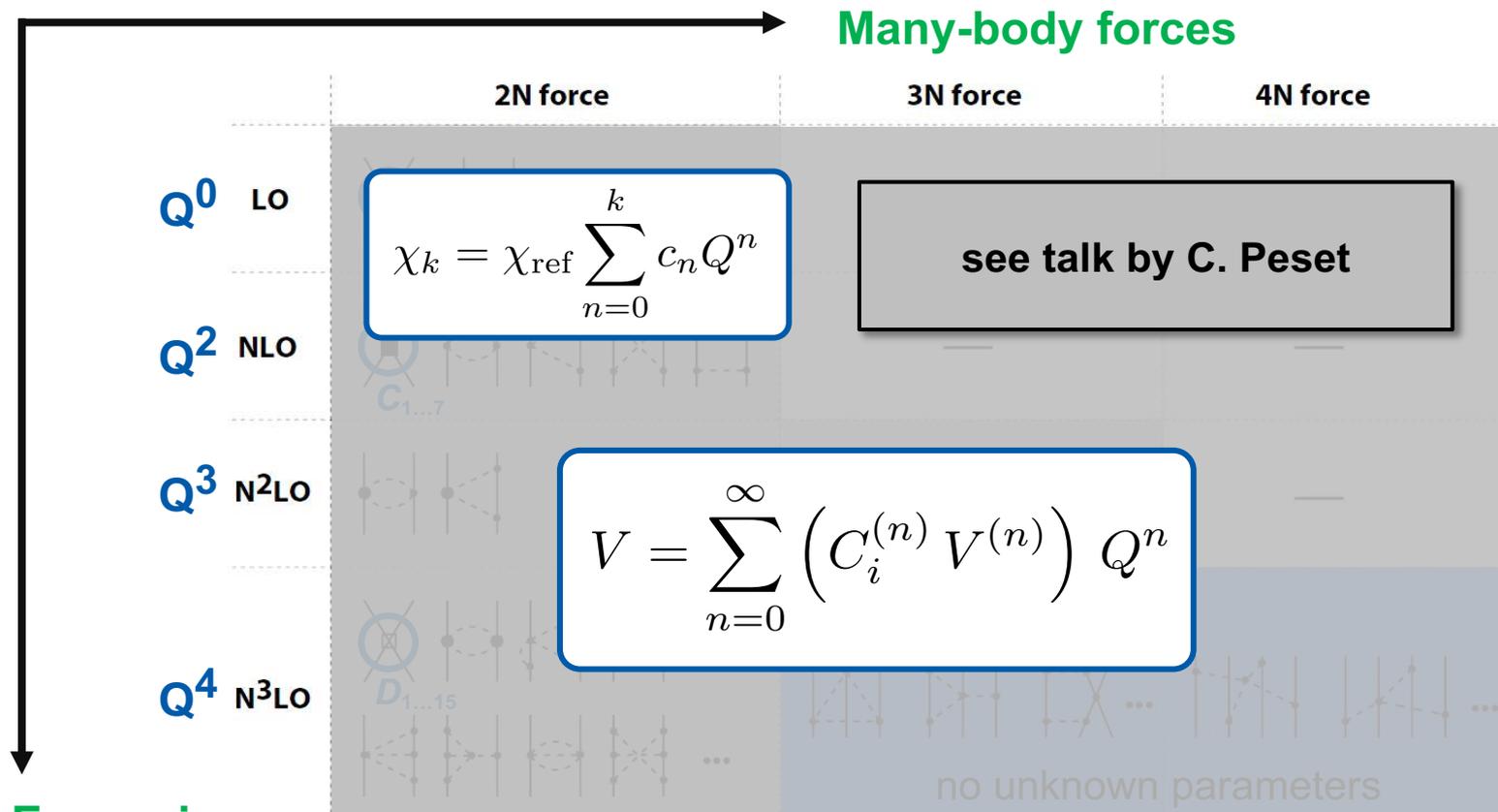
... and ongoing work at **N⁴LO** and even **N⁵LO**...

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Krebs, Machleidt, Meißner, ...

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Expansion

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Equation Of State

ground-state energy per particle of a system

$$\frac{E}{A} (n, \beta, T)$$

total density
neutron excess
temperature

consisting of **neutron** and **protons**

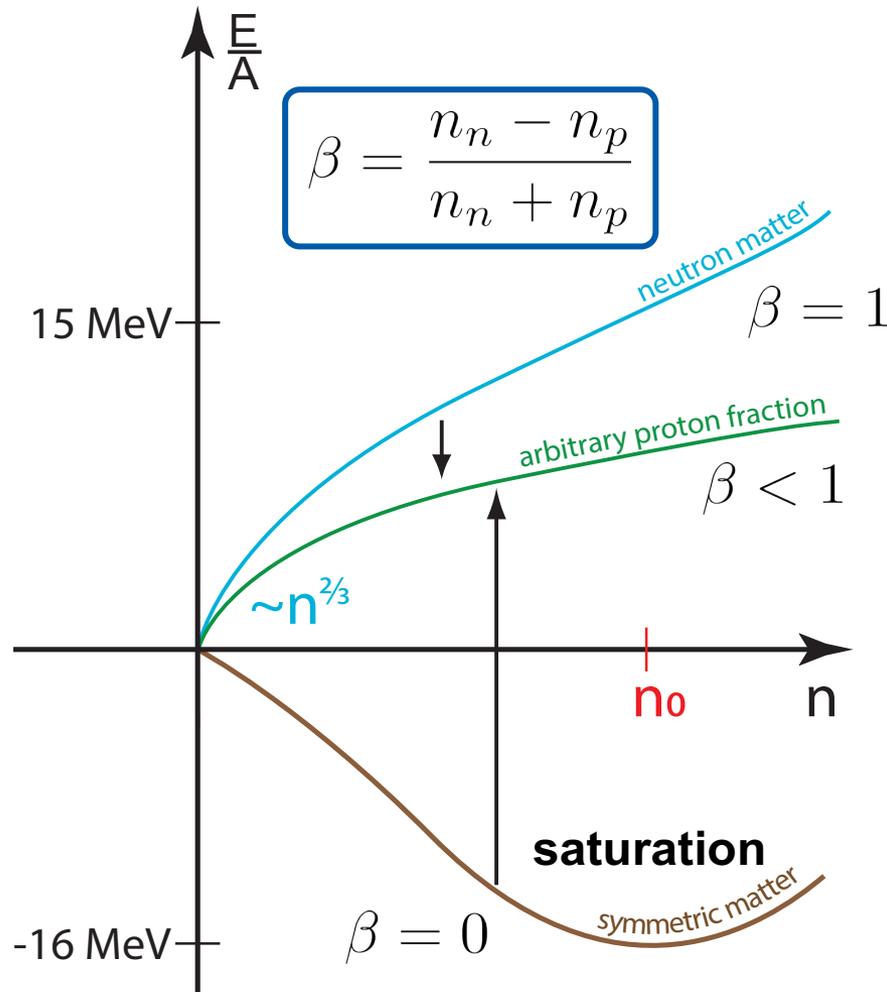
$$n = n_n + n_p$$

neutron | proton density

$$\beta = \frac{n_n - n_p}{n}$$

Recent progress in the nuclear-matter equation of state from chiral EFT

Homogeneous nuclear matter



theoretical **testbed** for nuclear forces with important consequences for EOS

saturation point

($n_0 \sim 0.16 \text{ fm}^{-3}$, $a_v \sim 16 \text{ MeV}$)

incompressibility ($K \sim 240 \text{ MeV}$)

symmetry energy ($E_{\text{sym}} \sim 32 \text{ MeV}$) and its **slope** ($L \sim 55 \text{ MeV}$) at n_0

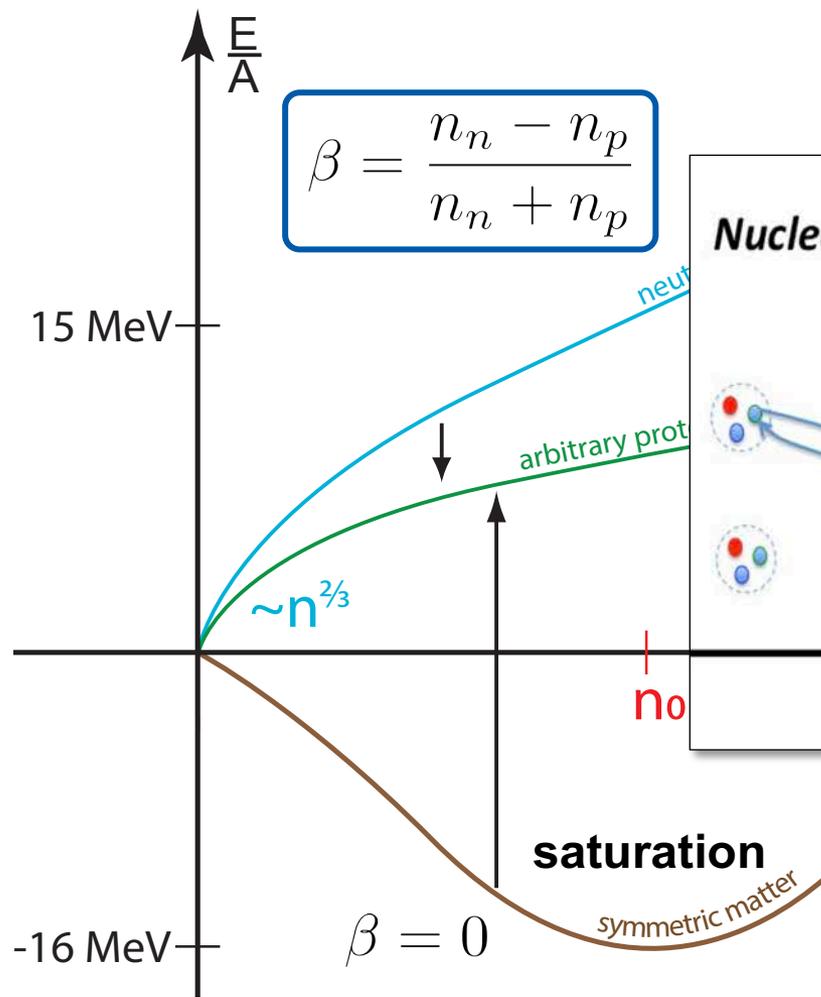
$$11.1 \text{ km} \leq R_{1.4 M_\odot} \leq 12.7 \text{ km}$$

Hagen *et al.*, Nat. Phys. **12**, 186

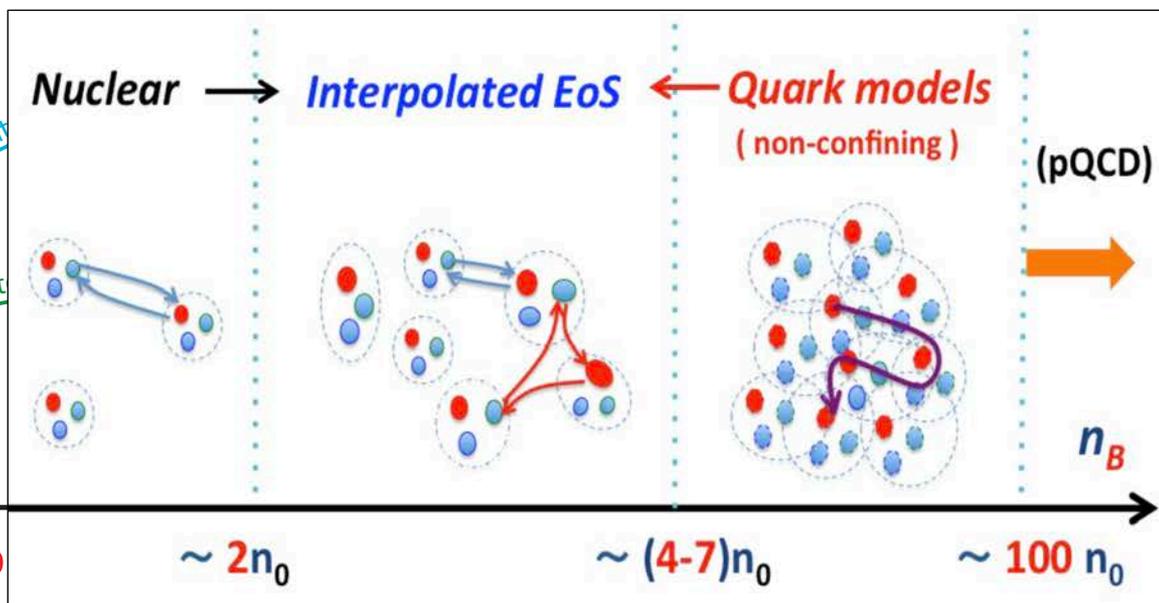
$$\frac{E}{A}(\beta, n) = \frac{E}{A}(\beta = 0, n) + \beta^2 E_{\text{sym}}(n)$$

Recent progress in the nuclear-matter equation of state from chiral EFT

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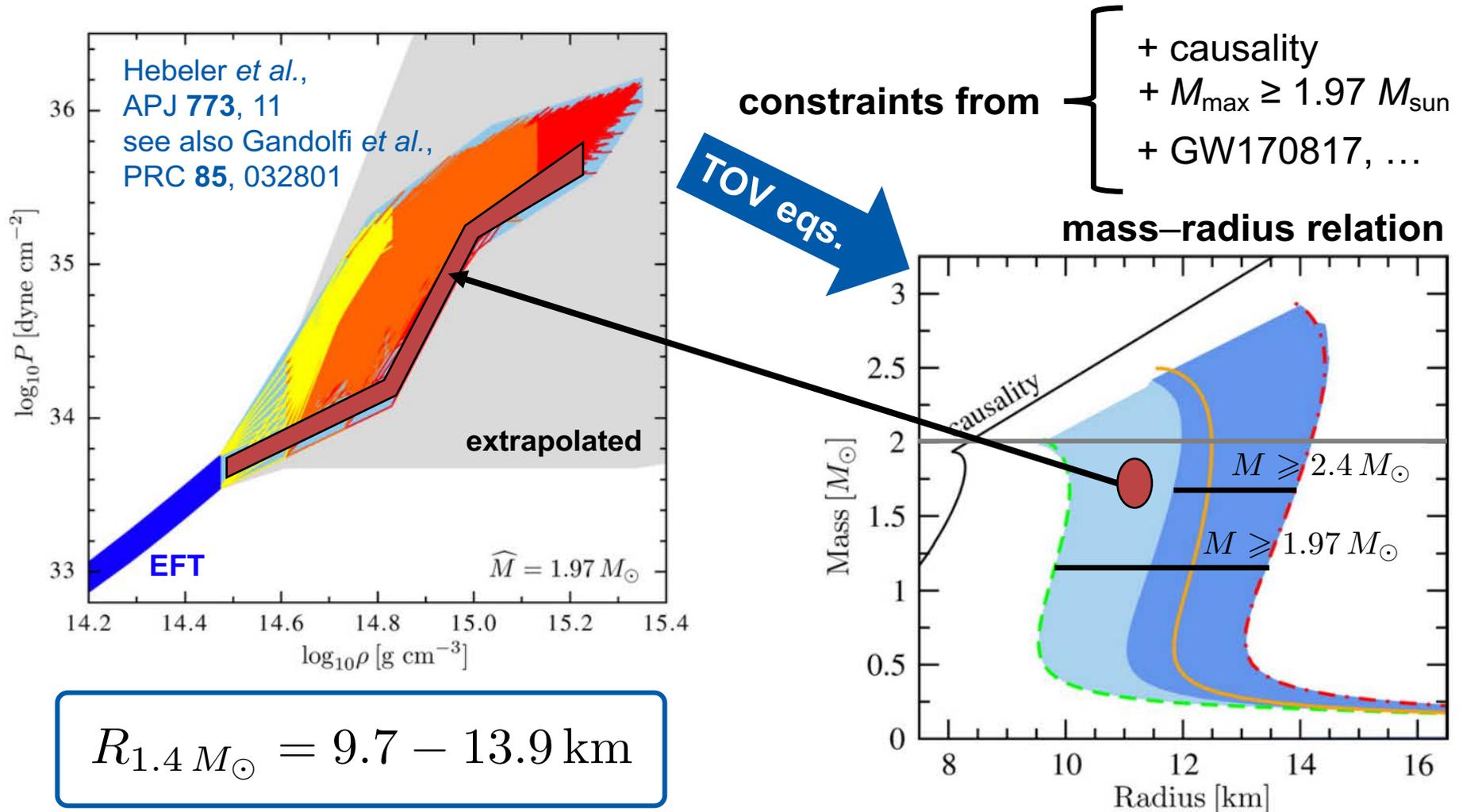
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$$\frac{E}{A}(\beta, n) = \frac{E}{A}(\beta = 0, n) + \beta^2 E_{\text{sym}}(n)$$

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Mass–radius relation

see, e.g., Greif *et al.*, MNRAS 485, 4



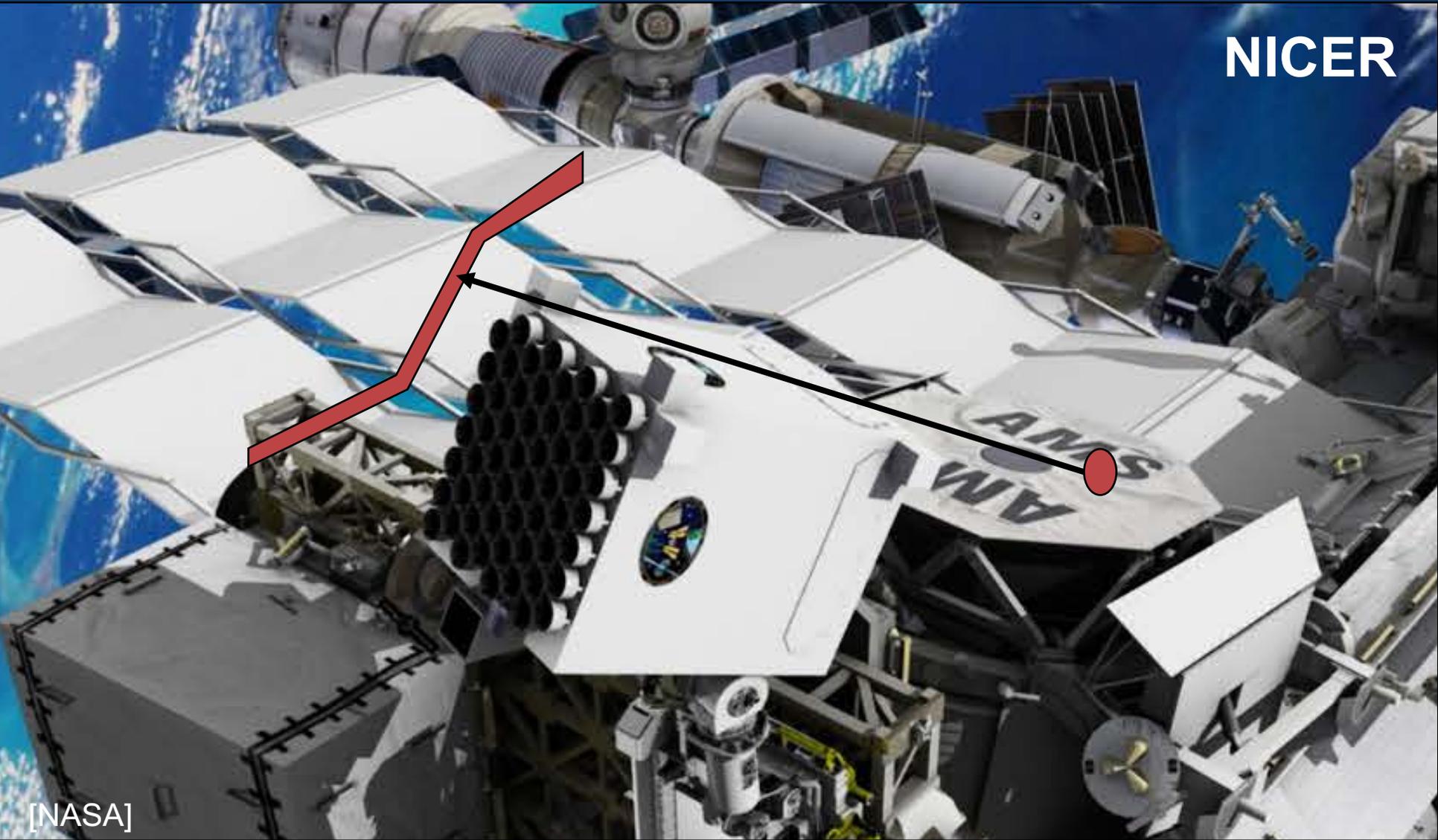
Recent progress in the nuclear-matter equation of state from chiral EFT

Mass–radius relation

Berkeley
UNIVERSITY OF CALIFORNIA

see, e.g., Greif *et al.*, MNRAS 485, 4

NICER

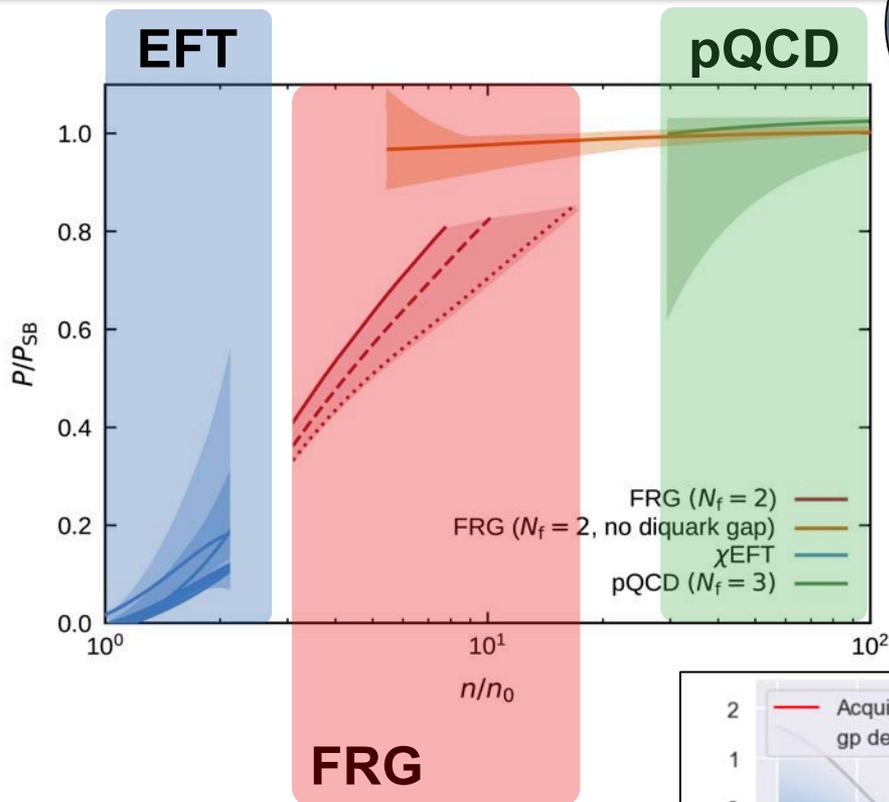


Recent progress in the nuclear-matter equation of state from chiral EFT

Connecting to QCD



Leonhardt, Pospiech, Schallmo,
Braun, CD, Hebeler, Schwenk,
arXiv:1907.05814

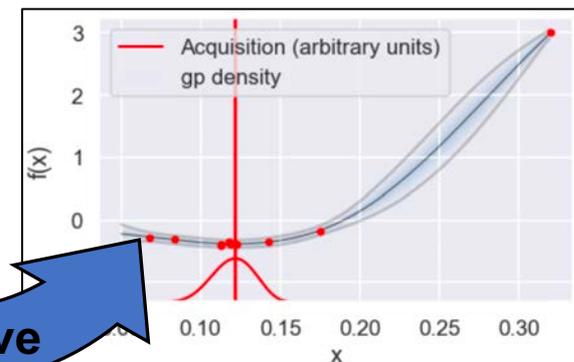
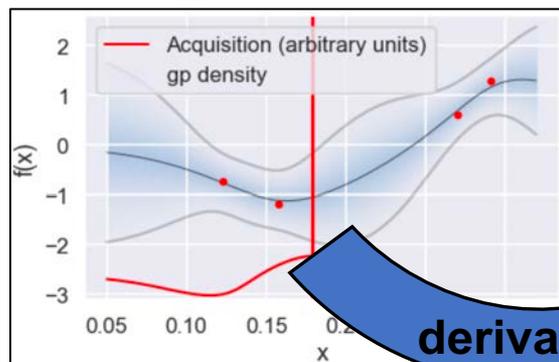


Pressure

$$P(n, \beta) = n^2 \frac{\partial E/A}{\partial n}(n, \beta)$$

EFT seems to **match** first
constraints from QCD at
intermediate densities

in future, derivatives
using GPs instead of
finite differencing?



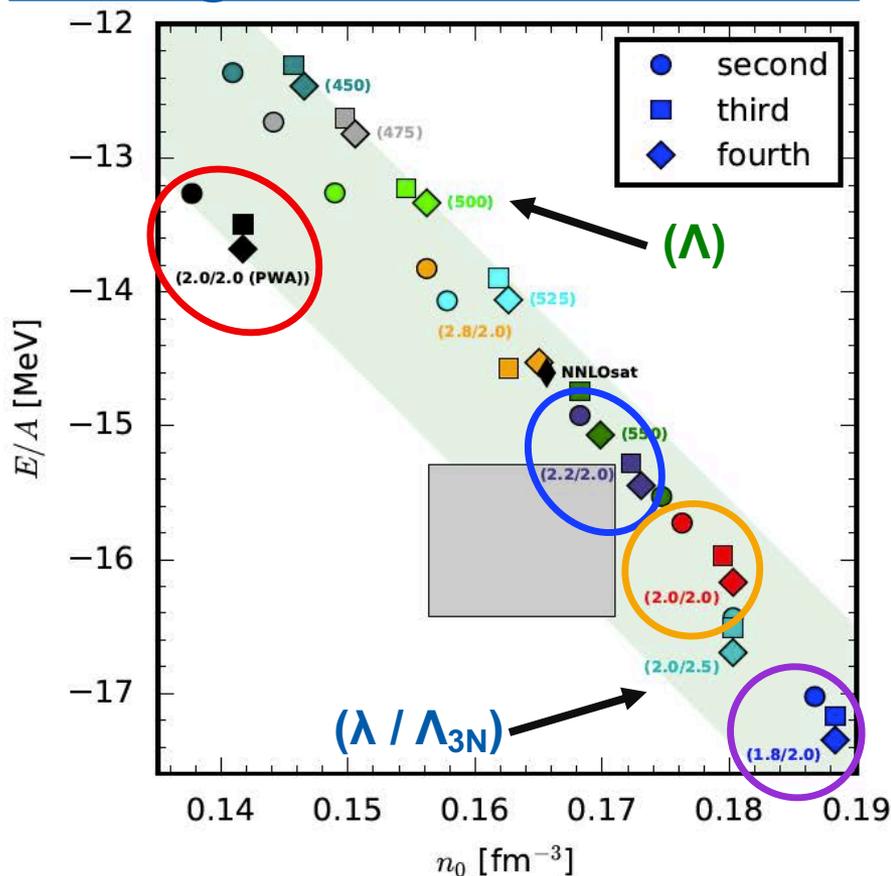
derivative

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Nuclear saturation

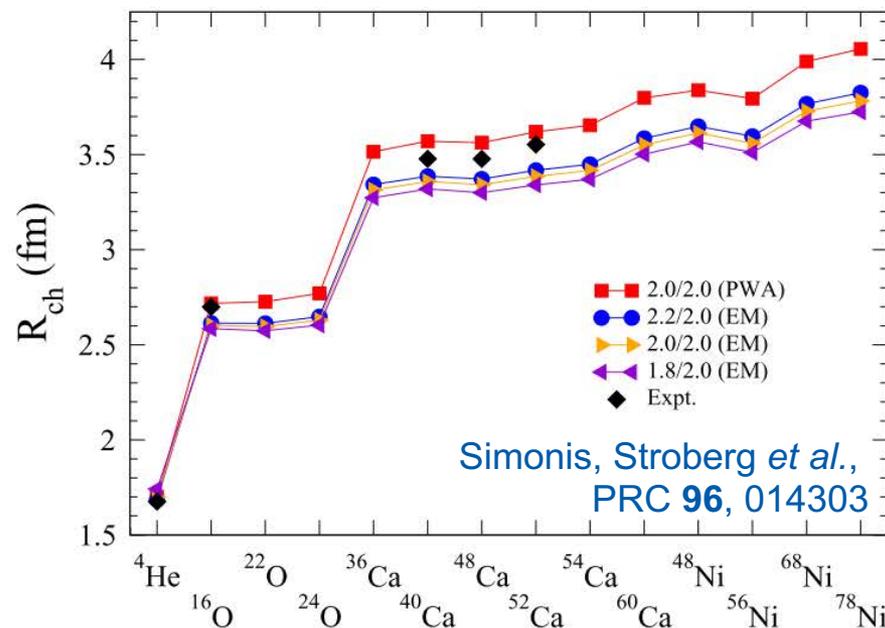
potentials: Hebeler *et al.*, PRC 83, 031301

Homogeneous Matter



magic 1.8 / 2.0 (EM) agrees well with experimental data!

Finite Nuclei



CD, Hebeler, Schwenk, PRL 122, 042501

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Efficient Monte Carlo framework

CD, Hebeler, Schwenk, PRL 122, 042501



efficient evaluation of **diagrams**
(single-particle basis)

- **implementing diagrams** has become **straightforward** (also particle-hole or $3N$ terms)
- using **automatic code generation** based on analytic expressions
- multi-dimensional momentum integrals: VEGAS (openMP, MPI, and CUDA)
- **computational beast**: controlled computation of arbitrary interaction or many-body diagrams

EOS up to high orders



automatic code generation



analytic form of the diagrams

Recent progress in the nuclear-matter equation of state from chiral EFT

Significant challenges!

CD, Hebeler, Schwenk, PRL 122, 042501



Higher orders: particle-hole contributions

Coraggio *et al.*, PRC 89, 044321; Holt, Kaiser, PRC 95, 034326



Exact normal-ordering

Holt *et al.*, PRC 81, 024002; Hebeler, Schwenk, PRC 82, 014314



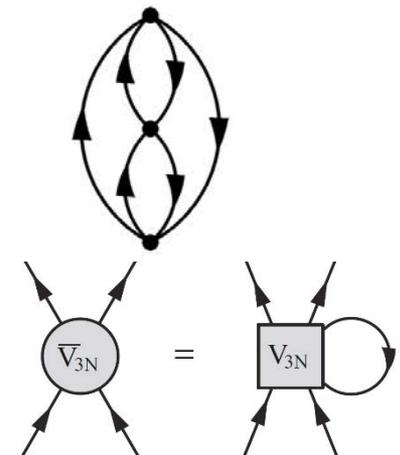
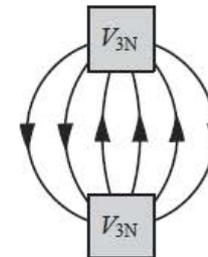
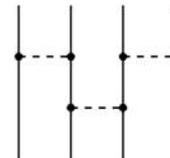
Residual 3N diagram included

Hagen *et al.*, PRC 89, 014319; Kaiser, EPJ A 48, 58



Higher many-body forces

Hebeler *et al.*, PRC 91, 044001



push **state-of-the-art** MBPT
calculations **to higher orders**

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Number of diagrams in MBPT

Stevenson, Int. J. Mod. Phys. C 14, 1135

The number of diagrams increases rapidly!

1, 3, 39, 840, 27 300, 1 232 280, ...

$n =$ 2 3 4 5 6 7

Integer sequence A064732:

Number of labeled Hugenholtz diagrams with n nodes.

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Number of diagrams in MBPT

Stevenson, Int. J. Mod. Phys. C 14, 1135

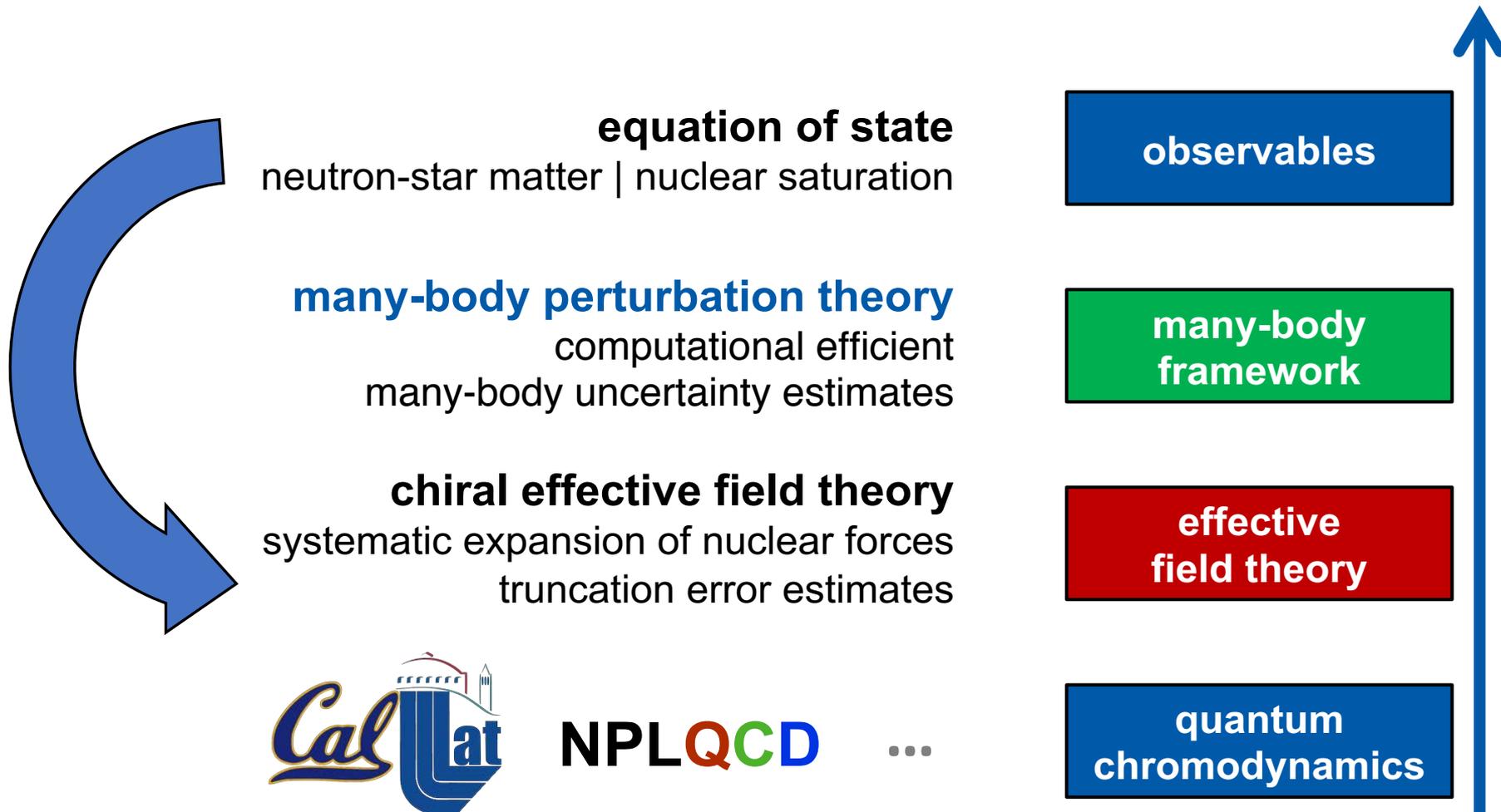
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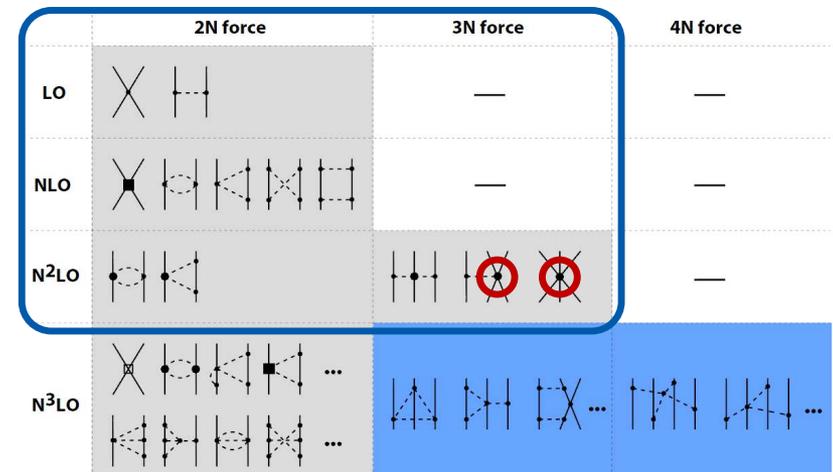
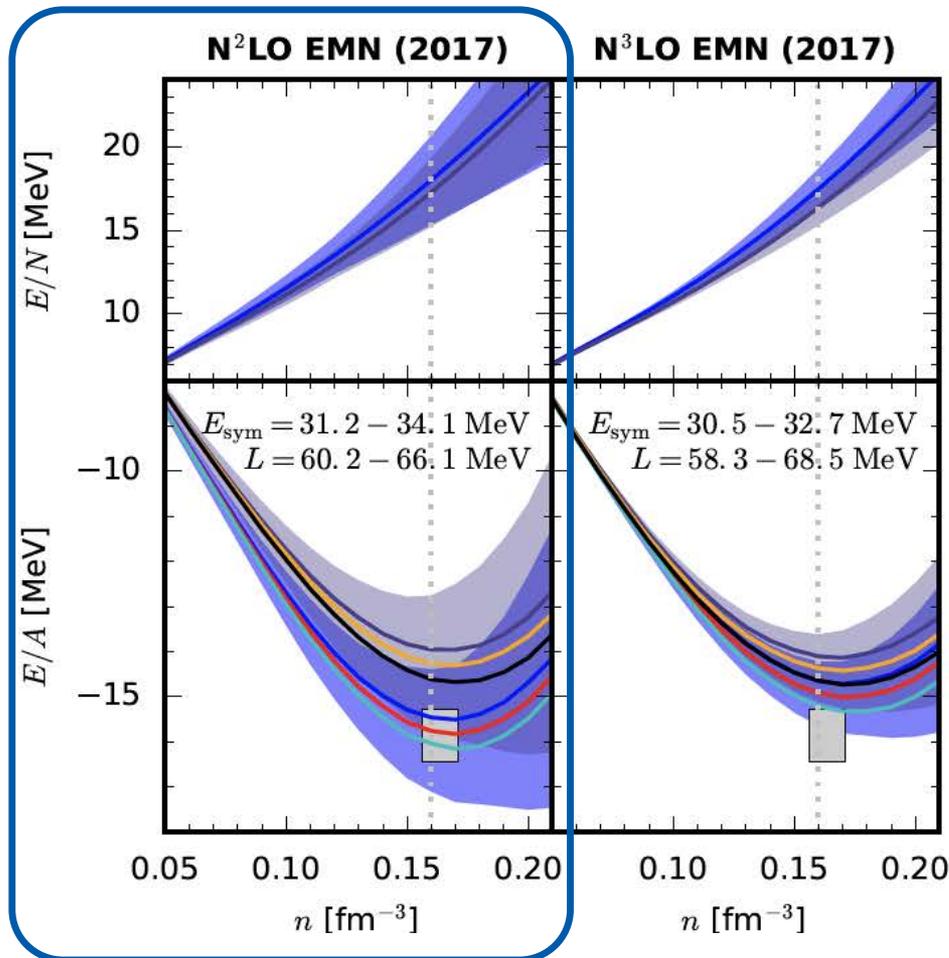
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Recent progress in the nuclear-matter equation of state from chiral EFT

Neutron and nuclear matter at N³LO

CD, Hebeler, Schwenk, PRL 122, 042501



reduced uncertainties
due to N³LO contributions !

left column:

Λ/c_D [MeV]/[1]	
— 450/2.25	— 500/ -1.75
— 450/2.50	— 500/ -1.50
— 450/2.75	— 500/ -1.25

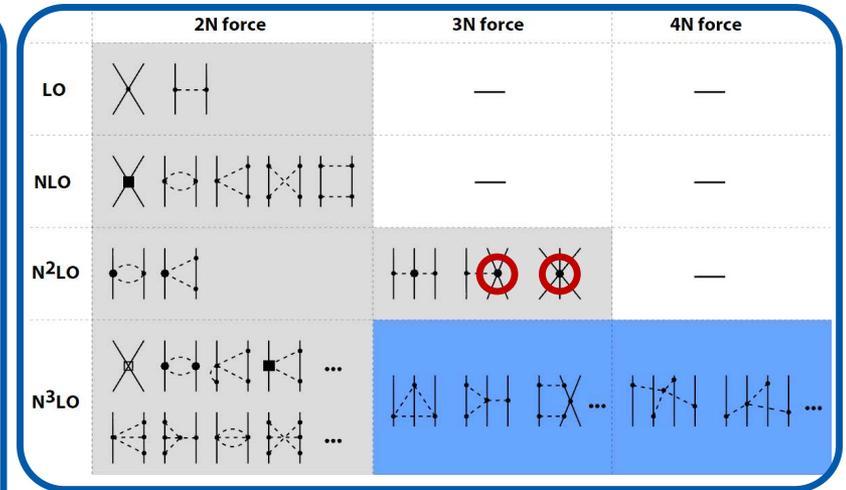
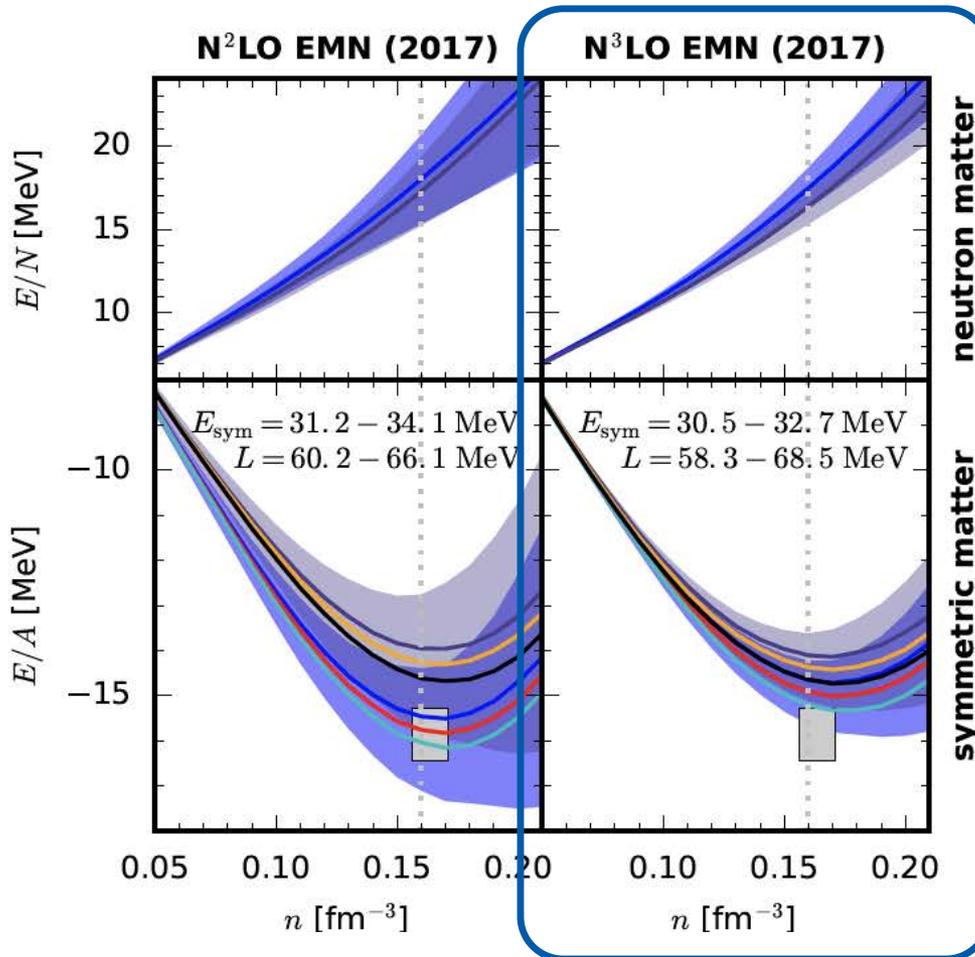
right column:

Λ/c_D [MeV]/[1]	
— 450/0.00	— 500/ -3.00
— 450/0.25	— 500/ -2.75
— 450/0.50	— 500/ -2.50

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Truncation error analysis



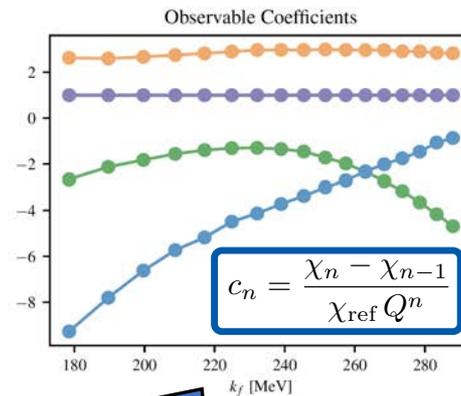
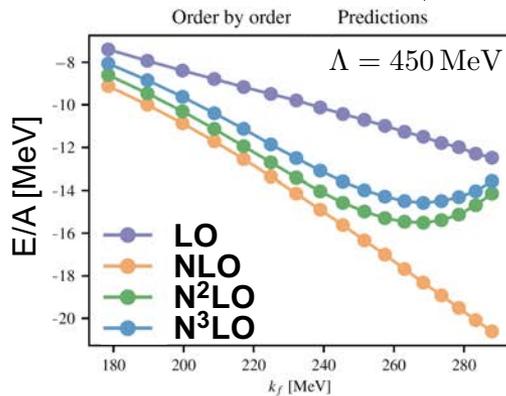
Melendez, Furnstahl, Phillips

start from chiral expansion for observable $X (= E / A)$

$$\chi^k = \chi_{\text{ref}} \sum_{n=0}^k c_n Q^n$$

Evaluate X at ...

... consecutive orders

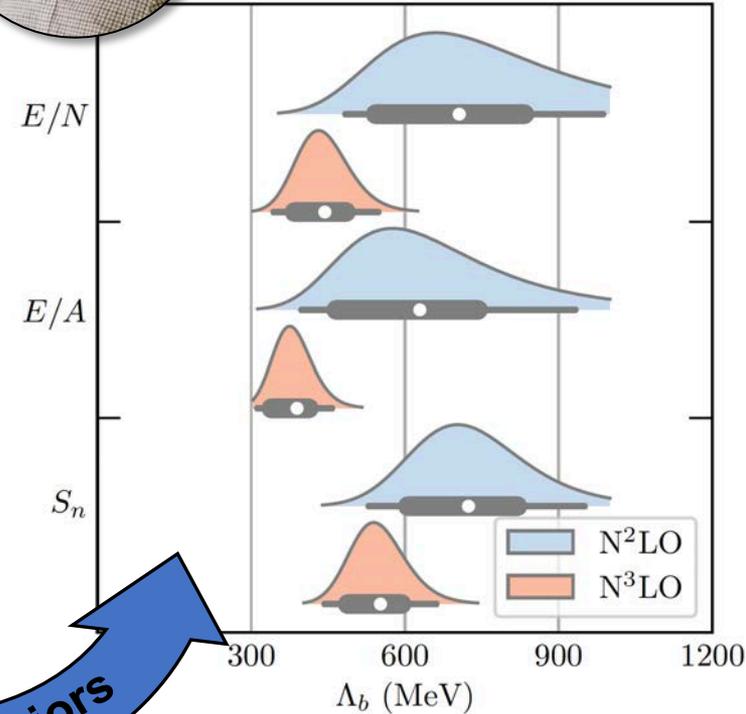


$$c_n = \frac{\chi_n - \chi_{n-1}}{\chi_{\text{ref}} Q^n}$$

check naturalness

GPs

$$Q = \max\left(\frac{p}{\Lambda_b}, \frac{m_\pi}{\Lambda_b}\right)$$



priors

Bayesian inference: breakdown scale, higher-order terms

BUQEYE collaboration; Melendez *et al.*, arXiv:1904.10581, Melendez, Wesolowski *et al.*, PRC **96**, 024003, ...

Recent progress in the nuclear-matter equation of state from chiral EFT

Summary and outlook

- 1 Perform zero-T calculations (up to high order)**
resummation, higher-order single-particle spectra, ...
- 2 Work out finite-T extension (to third order)**
finish developments, study thermodynamic properties, ...
- 3 Construct high-density | temperature EOS**
observational constraints, interface to astrophysics, ...
- 4 Quantify theoretical uncertainties**
Bayesian truncation errors: naturalness, breakdown scale, ...

Collaborators:

R. Furnstahl

J. Melendez

D. Phillips

K. Hebeler

K. McElvain

A. Schwenk

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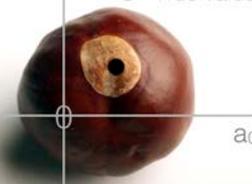


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ENERGY

a_1
■ Prior
■ Posterior
● True value



BUQEYE Collaboration